HD1761
G6
No.328

S

3 1175 00566 2518

STACKS

DIVISION OF AGRICULTURAL SCIENCES
UNIVERSITY OF CALIFORNIA

LONG-TERM CONTRACTING STRATEGIES FOR AGRICULTURAL PROCESSING FIRMS WITH PARTICULAR REFERENCE TO FARMER COOPERATIVES

By
Steven T. Buccola &
Ben C. French

UNIVERSITY OF CALIFORNIA
DAVIS
MAY 1[illegible] 1979
SER. REC. LIBRARY

CALIFORNIA AGRICULTURAL EXPERIMENT STATION
GIANNINI FOUNDATION OF AGRICULTURAL ECONOMICS

Giannini Foundation Research Report No. 328

December, 1978

University of California, Davis
Department of Agricultural Economics

Long-Term Contracting Strategies for Agricultural Processing Firms with Particular Reference to Farmer Cooperatives

by

Steven T. Buccola and Ben C. French*

---

*Steven T. Buccola is Assistant Professor, Department of Agricultural Economics, Virginia Polytechnic Institute and State University, and Ben C. French is Professor, Department of Agricultural Economics, University of California, Davis.

ACKNOWLEDGMENTS

This study was financed under a research agreement between the California Agricultural Experiment Station and the Economics, Statistics, and Cooperatives Service, Office of the Deputy Director for Cooperatives (formerly Farmer Cooperative Service), U.S. Department of Agriculture. We owe particular thanks to Jack Armstrong of ESCS who acted as our liaison during the study period.

Thanks are also due Don Schulak and Tom Jensen of Tri/Valley Growers, San Francisco, who provided much of the information necessary to development of this report.

We appreciate the assistance of Lee Garoyan, Sam Logan, and Jerry Nishimoto of the Department of Agricultural Economics, University of California, Davis, during the early phases of the analysis and the very helpful criticisms and suggestions of two anonymous Giannini Foundation reviewers in the process of preparing the final manuscript.

TABLE OF CONTENTS

Page

I. INTRODUCTION. 1
Research Problem and Objectives 1
The Special Nature of Farmer Cooperative Contracting. . . . 3
Alternative Price Clauses of Contract 5

II. STRUCTURE OF THE THEORETICAL AND EMPIRICAL MODELS 12
Decisions Under Risk. 13
The Profit Functions. 16
Defining the Cooperative Cost-Plus Sales and Sales-Minus Purchase Options. 17
The Expected Acreage Requirement. 25
The Future Profit Stream. 26

III. ESTIMATION OF EXPECTED VALUES, VARIANCES, AND COVARIANCES . 28
Projection Methods. 28
Expected Values and Variances 28
Covariances 29
Converting Moments from a Tonnage to an Acreage Basis . . 31
Empirical Estimates 32
Historical Data Series. 33
Historical Statistical Analyses and Estimates of K, $\mathrm{Var}(E_t)$ 40
Estimates of Subjective Trend Probabilities p(B). 42
Calculation of Ten Year Probability Moments 43

IV. EFFICIENT PORTFOLIOS OF CONTRACTUAL ARRANGEMENTS. 50
Grower Efficiency Frontiers 51
Cooperative Efficiency Frontiers. 57
Distributor Efficiency Frontiers. 66
Relative Risk and the Task of Interpreting Efficiency Frontiers 72

V. ESTIMATION OF MONEY UTILITIES 75
Utility Measurement 76
Interview Method. 77
Interview Results 81
Estimates of Utility Functions. 88
Quadratic Functions 89
Exponential Functions 91
Quadratic and Exponential Functions Compared. 92

VI. OPTIMAL CONTRACT PORTFOLIOS AND CONTRACT EQUILIBRIUM. . . . 94
Problem Specification 94
Optimal Choice Under Risk Neutrality. 97
Optimal Choice Under Risk Aversion. 101
Selection of Representative Utility Functions 103
Equivalent Annual Rates of Present Value Profit Sums. . . 105
Relation of Portfolio Shares to Contract Parameters . . . 106
Contract Equilibrium. 121
Thin Market Solutions 124

Page

VII. SUMMARY AND CONCLUSIONS. 126
Summary of Research Procedures 126
General Conclusions. 131
  Impact of Degree of Risk Aversion. 131
  Role of the Open Market. 132
  Suggestions for Self-Analysis. 134
  Further Research Needs 135

APPENDIX A 137
Grower 137
Cooperative. 139
Distributor. 141
Expected Utility Problems. 142

APPENDIX B
Optimizing Solutions for the Expected Utility Functions. . 144
A. Grower solution. 144
  1. Expected utility function - quadratic utility. . . 144
  2. First order conditions for maximum 145
  3. Solutions $[0 \leq S_1, S_2, .25-S_1-S_2 < .25]$. 146
    (1) Interior solution $[0 < S_1, S_2, .25-S_1-S_2 < .25]$ 146
    (2) Boundary solutions. 146
B. Distributor and processor solutions. 147

APPENDIX C
Procedures for Constructing Historical Series of Cost Variables. 149
Tomato Farm Inputs (VCF^a_{tom}). 149
  Labor. 149
  Seed, Pesticides, and Fertilizer 157
  Diesel and Water 158
  Land Rent. 158
  Total Costs. 159
  Labor. 161
  Drum 161
  Electricity and Gas. 161
  Tomato Hauling 164
  Total Cash Costs 164

APPENDIX D
Procedures for Constructing Historical Series of Revenue Variables. 165
U.S. Wholesale Prices of Consumer-Size Tomato Sauce (MP^{tr}_{sce}). 165
U.S. Wholesale Prices of California Bulk-Packed Tomato Paste (MP^{tr}_{pst}). 167
Nontomato Farmer Revenue ($REVF^a_{ntom}$). 170
Note on Price Deflation. 173

REFERENCES 174

LIST OF TABLES

Table		Page
1.1	Alternative Pricing Formulae for Long-Term Marketing Contracts	6
2.1	Grower Profit Function	18
2.2	Cooperative Net Margin Function	19
2.3	Distributor Profit Function	21
2.4	Definitions of Terms for Tables 2.1 through 2.3	22
3.1	Annual Series of Revenue and Cost Variables Specified in Profit Functions of the Cooperative Processor and Growers, 1951-1974, 1974 Dollars	34
3.2	Summary of Results from Time Trend Fits to Deflated Revenue and Cost Variables	41
3.3	Calculation Procedures to Obtain Estimates of Probability Moments of Future Linear Trends (B), Revenue and Cost Variables, 1975-1984	44
3.4	Estimates of Ten-Year Sums of Discounted Means and Variances: Yields, Revenue and Cost Variables	46
3.5	Means and Variances of Present Value Sums of Profit Variables Expressed on a Per-Acre Basis	47
3.6	Prediction Correlation Coefficients and Covariances Among Revenue and Cost Variables	49
4.1	E-V and Coefficient of Variation Frontiers Indicating Efficient Portfolios of Market Price, Cost-Plus, Sales-Minus, and Cooperative Member Sales Contracts for a California Producer of Processing Tomatoes	53
4.2	Parameter Changes Associated with Cooperative Efficiency Frontier Sets	58
4.3	E-V and Coefficient of Variation Frontiers Indicating Efficient Portfolios of Market Price, Cost-Plus, and Sales-Minus Contracts for Cooperative Tomato Paste Sales and Raw Tomato Purchases	62
4.4	Parameter Changes Associated with Distributor Efficiency Frontier Sets	67
4.5	E-V and Coefficient of Variation Frontiers Indicating Efficient Portfolios of Market Price, Cost-Plus, and Sales-Minus Purchase Contracts for a Reprocessor of Bulk Tomato Paste (Distributor)	68
5.1	Summary of Grower Socioeconomic Data	82
5.2	Grower Responses to Utility Function Questionnaires	83
5.3	Responses to Utility Function Questionnaires: Cooperative Spokesmen	85
5.4	Estimates of Quadratic Money Utility Functions, $U = a + bM + CM^2$	90
6.1	Observations on Grower's Contract Portfolio Offer Curves for Sale of Tomatoes, Nonmember Options Only	109
6.2	Observations on Cooperative Contract Portfolio Offer Curves for Tomato Paste Sales and Tomato Purchases, Assuming Quadratic Money Utility	110
6.3	Observations on Distributor Contract Portfolio Offer Curves for Tomato Paste Purchases	111

Table | Page

6.4 OLS Approximations of Grower Offer Curves 112
6.5 OLS Approximations of Processor Offer Curves. 113
  1. Sales side. 113
  2. Purchase side 114
  3. Interaction 114
6.6 OLS Approximations of Distributor Offer Curves. 115

Appendix Table

C1 Annual Labor Hours Per Acre to Cultivate and Harvest Processing Tomatoes in California, Selected Counties, by Operation and Wage Class. 150
C2 Fuel and Repair Costs Associated with Producing Processing Tomatoes in California, 1970-1973, by Selected County, Including Procedure to Compute Repair Labor Hours Per Acre. 151
C3 Input Units Required Annually to Produce One Acre of Processing Tomatoes in California, by Selected County, 1970-1973. 152
C4 Calculation Procedure Used to Obtain Weighted Average Hourly Wage Paid to Workers on Processing Tomato Farms, Lower Sacramento and Upper San Joaquin Valleys, California, 1973. 154
C5 Calculation Procedure Used to Obtain Gallons of Diesel Required Annually in Producing an Acre of Processing Tomatoes, California, 1970-1973, Selected Counties. 155
C6 Per-Acre Input Coefficients and Annual Unit Input Prices, Processing Tomato Farms, California, 1951-1974. 156
C7 Calculation Procedure Used to Obtain Per-Acre Land Rent Estimates for Processing Tomato Farms, California, 1951-1974, Owner-Operators and Share-Lessees. 160
C8 Summary of Annual Per-Acre Costs to Produce Processing Tomatoes, Weighted Average of Selected Six-County Area, California, 1951-1974, Owner-Operator and Share-Lessee, Assuming 1973 Production Technology. 162
C9 Indexes of Input Prices and Total Non-Raw-Product Costs to Process Raw Tomatoes into Bulk-Packed Tomato Paste (32 percent), California, 1951-1974, Assuming 1974 Production Technology. 163
D1 Procedure for Calculating Tomato Sauce Revenue of the Distributor/Reprocessor 166
D2 Annual Average Tomato Paste Prices, California, 1951-1974, by Container Size and Percent Solids. . . 168
D3 Computation Procedure Used to Estimate Average Annual Market Value of Bulk-Packed Tomato Paste Produced from One Acre of Processing Tomatoes, California, 1951-1973. 171
D4 Average Annual Prices, Yields, and Per-Acre Revenues Earned from Corn, Wheat, and Dry Edible Bean Cultivation, California, 1951-1974 172

Figures | Page

4.1 Grower's Efficiency Frontiers (σ_π^2 = variance of profit in thousand trillions, $\frac{\sigma_\pi}{\mu_\pi}$ = coefficient of variation) 54

4.2 Cooperative's Efficiency Frontiers (σ_π^2 = variance of profit in thousand trillions, $\frac{\sigma_\pi}{\mu_\pi}$ = coefficient of variation) 60

4.3 Distributor's Efficiency Frontiers (σ_π^2 = variance of profit in thousand trillions, $\frac{\sigma_\pi}{\mu_\pi}$ = coefficient of variation) 69

5.1 Money Utility Observations for the Grower Sample. . . . 84

5.2 Money Utility Observations for the Cooperative Executive and Director. 85

6.1 Maximum Expected Profit Choices for a Grower with 75 Percent of Production Allocated to the Cooperative on a Profit-Share Basis, the Remainder Either Cost-Plus, Sales-Minus or Market Price. AAF = 1210, OAFF = 1940. 98

6.2 Maximum Expected Net Revenue Contract Choices for the Cooperative Assuming the Grower Cost-Plus Markup is k = 1.42, the Sales-Minus Markdown ℓ is .396 and ACC = 18,860. 100

6.3 Maximum Expected Profit Choices for the Distributor . . 102

6.4 Regions of Optimal Contract Portfolios for the Grower Sales Alternatives 118

6.5 Regions of Optimal Contract Portfolios for the Cooperative Sales Alternatives (k = 1.35, ℓ = .39). 119

6.6 Regions of Optimal Contract Portfolios for the Distributor Sales Alternatives. 120

I. INTRODUCTION

Research Problem and Objectives

An important deterrent to capital expansion, and to business activity generally, is uncertainty associated with net returns. Uncertainty may be attributed to variations in prices or demand for a firm's output; variations in prices, supplies or quality of inputs; unforeseen changes in the technical factors governing conversion of input to output; or unpredictable government intervention in output or input markets. A firm will view its total business uncertainty as a combination of these several variables measured, for the most part, by the historical experience of the industry as augmented by a prognosis of future conditions.

In the past half century, there has been a strong trend toward the use of long-term or annual sales contracts between farm producers and processors as a means of reducing variability of prices and shipment volumes and of controlling quality [Mehren]. This trend may benefit both processors and farmers by stabilizing costs of the former and sales revenue of the latter. Examples of such contractual situations are farmer-member agreements with processing cooperatives, agreements between individual farmers and proprietary processors, and contracts signed by bargaining cooperatives with proprietary processors on behalf of or in conjunction with member farmers.

Cooperative and proprietary processors have in the past decade taken an interest in contracting the sale of their processed output to distributors and retailers. Supplying food retailers with private label goods has undoubtedly stimulated this trend, since private label goods must be processed according to quality requirements specified by the retailer. But sales contracts are being used over an increasing variety of semi-processed and finished agricultural commodities [Garoyan].

The trend toward contract sales in U.S. agriculture is a result of several factors, including: the decline of central markets; increasing concentration of firms at all levels; the growing demand of consumers for standardized products, especially as they apply to convenience goods and institutional supplies; and the economic unsuitability of input fluctuations to heavily capitalized processing and distribution equipment. Increasing market concentration and capitalization have been especially strong stimulants to contracting. Fewer buyers and sellers usually mean less security for both regarding volumes of purchases and sales; and larger indivisibilities in processing/marketing equipment make short-term economic adjustments more difficult for managers. The increasing proportion of agricultural goods shipped under long-term contract, in turn, further impairs competitive conditions by reducing the importance of free market sales. This snowball effect makes the determination of market price and equilibrium conditions of exchange increasingly difficult.

Marketing contracts may be written in many ways. They may include, in addition to price and quantity considerations, specification of shipment timing, quality and grade, packaging, transportation arrangements, payment timing, dockage terms, credit, technical advice, liability limitations, delinquency damages, negotiation stipulations, renewability options, tie-ins with other contracts, and conditions under which contracted terms will be amended. In formulating its sales and/or purchase strategies, the individual firm must not merely choose between open market and contract sales, but must choose among the open market and a wide variety of contract instruments. More generally, the optimal choice may involve a combination or portfolio of sales methods.

In order to evaluate alternative possibilities, a firm must consult its own set of business objectives, its willingness to take risks, and the technical and financial resources to which it has access. However, the firm's initial choice does not end the contracting process. Mutual agreement must be forged between the contracting parties, each of whom may have widely differing business objectives and proclivities for risk-taking, and each of whom may be expected to try to win the most advantageous conditions. Economic analysis of alternative contract strategies thus must consider not only the basis for optimal choice of the individual firm, but the manner in which terms of trade may be varied to achieve final agreement between the contracting parties.

The objectives of this study are (a) to develop a general framework which, we hope, will aid agricultural processing firms in selecting and evaluating long-term contracting strategies, and (b) to suggest how, under restricted circumstances, equilibrium solutions to contracting negotiations may be achieved. The study focuses on alternative ways of specifying product pricing provisions under conditions of uncertainty and aversion to risk. The model is applied to a California fruit and vegetable processing cooperative which contracts backward with member and nonmember growers for tomatoes and forward with buyers of the coop's tomato paste. The results show how optimal contract choices for producers, processors, and distributors may be affected by various attitudes toward risk and the parameter values of alternative pricing formula.

The Special Nature of Farmer Cooperative Contracting

The financial situation presented by processing cooperative contracting is unique in U.S. business. Grower-members are owners of their own processing facilities; the cooperative has moral and legal responsibility for paying them on some basis of processor returns.

Since conditions in the processed product market, including the character of contracts entered into on that market, affect processor profits, returns to grower-members are directly affected by sales arrangements made by the cooperative for processed product.

Cooperatives may also experiment with alternative contract terms governing purchase of raw products from nonmembers. But there are legal restrictions that affect the price clauses of nonmember contracts. Cooperatives claiming tax exempt status under Internal Revenue Code S.521 must compensate members and nonmembers alike on a patronage, that is, profit share basis [Touche Ross, p. 50]. Even nontax exempt cooperatives must purchase not less than one-half of their raw product from member suppliers, and thus on a profit share basis. The remaining purchases may be conducted on any terms buyer and seller agree provided trade discrimination laws are not violated. Grower-member returns are directly affected by the character of nonmember purchase agreements just as they are by processed product sales agreements.

Obligations between cooperatives and their membership provide another application of contract analysis. Important issues are size of membership, distribution of acreage or tonnage allocations among members, the nature of advances paid to member-growers at delivery, provisions for secondary pools, bases for establishing each member's pro-rata share of cooperative net margins, and retain provisions. Many advance payments to members, for example, are fractions of a prevailing market price. But such prices are not always available or meaningful, and in any event members may prefer a more stable mechanism.

Although an interesting and important element of cooperative organization, membership contracts are not explicitly considered in the present report. It is assumed instead that a cooperative is a unified body with an interest in its terms of trade with customers and nonmember suppliers. A primary purpose of processing cooperatives is to provide a stable return to members' produce; thus special attention is paid to the stability of cooperative net margins over time, and the influence of alternative market contracts on this stability.

Alternative Price Clauses of Contract

To formulate an optimal sales or purchase strategy, it is necessary first to explore alternative ways of specifying the major contract provisions. In this section, we shall describe briefly a range of pricing options which have been used in practice or suggested as theoretically appealing. Some of these ideas are applicable to a severely restricted set of institutional conditions, whereas others have potentially widespread use. In Table 1.1, selected pricing options are organized into nine generic categories, with brief summaries of the expected advantages and disadvantages associated with each category.

The fixed price, market price, cost-plus, and sales-minus price formulae are discussed in J. Dean, Hirschleifer, and elsewhere, mainly in regard to intra-company pricing. However, they have a broad appeal to inter-firm pricing as well. Cost-plus contracts become especially popular in inflationary periods. Sales-minus contracts constitute a revenue share and are often used by cooperatives under the title "secondary pool."

The inventory model under the market price proxy category is taken from Chiang, (p. 520); another proxy, the inflation escalator, is often employed in government contracts for expensive hardware.

TABLE 1.1 Alternative Pricing Formulae for Long-Term Marketing Contracts

Pricing formula[a]	Description	Advantages[b]	Disadvantages[b]
Fixed or negotiated price (a) $P = a_0$ (b) $P = a_1 + b_1t$.	Buyer pays seller a fixed and stated price. (a) Price may be invariant over time. (b) Price varies over time t.	Eliminates price uncertainty. May reflect competitive conditions if buyer and seller are equally powerful negotiators.	Does not relate to a supply or demand variable, nor to any measure in the light of which both parties may consider P "fair."
Market price $P = MP$.	Buyer pays seller a market price MP defined as to time period, place and product specifications.	Eliminates possibility of lost market opportunity for either party.	Market price may not exist, represent a small proportion of transactions, or be highly unstable.
Market price proxy	Buyer pays seller a price determined by value of some agreed economic indicator. For example	May be used to "simulate a market" if none exists.	Requires extensive statistical analysis or significant assumptions to verify the market model.
(a) $P = a_2 + b_2$ CPI	(a) Price may vary with consumer price index CPI.	(a) Stabilizes buyer and seller purchasing power.	(a) Poorly related to any specific industry conditions.
(b) $P = a_3 - b_3I$.	(b) Price may vary with inventory level I of buyer or seller.	(b) Stabilizes buyer and seller inventories; has theoretical merit as price determinant.	(b) Does not explicitly consider current production or future supply/demand expectations.
Cost-plus	Buyer pays seller the latter's unit production cost UCS plus a premium. Cost may be specified to include total cost or only variable cost. Price may be determined by:		May discourage seller from making cost-saving technical changes. Seller may pad costs.
(a) $P = UCS + K, K > 0$	(a) Adding a constant K to costs.	(a) Guarantees seller a fixed gross margin.	
(b) $P = k \cdot UCS, k > 1$.	(b) Multiplying costs by a number k greater than one.	(b) Guarantees seller a rate of gross margin over cost.	

(table continued)

TABLE 1.1 continued

Pricing formula[a/]	Description	Advantages[b/]	Disadvantages[b/]
Sales-minus	Buyer pays seller a portion of the commodity's unit resale value URB. Price may be determined by:		May discourage buyer from vigorous marketing efforts.
(a) $P = URB - L,\ L > 0$	(a) Subtracting a constant L from the buyer's unit resale value.	(a) Guarantees buyer a fixed return over the indicated input.	
(b) $P = (\ell \cdot URB),\ 0 < \ell < 1.$	(b) Multiplying the buyer's unit resale value by a constant ℓ less than one.	(b) Guarantees buyer a rate of return over the indicated input.	
Profit share	Buyer pays seller a portion of net unit profits URB-UCB earned from the commodity's resale. Profit calculation may include or exclude fixed expenses. Price may be determined by:		May discourage buyer from vigorous profit seeking behavior.
(a) $P = (URB-UCB) - Z,\ Z > 0$	(a) Subtracting a constant Z from the buyer's unit profit.	(a) Buyer transfers entire profit risk to seller, an advantage if buyer risk in other activities is already near psychological limit.	
(b) $P = (URB-UCB)z,\ 0 < z < 1.$	(b) Multiplying buyer's unit profit by a constant z less than one.	(b) Buyer transfers part of profit risk to seller, an advantage if buyer is not willing to bear all additional profit risk.	

(table continued)

TABLE 1.1 continued

Pricing formula[a]	Description	Advantages[b]	Disadvantages[b]
Market price, cost-plus combination[c]	Buyer pays seller cost-plus unless this differs too much from the market price, in which event a compromise price is calculated.[d] Maximum cost-plus, market price difference and/or compromise price may be calculated on a constant or proportionate basis. Some combinations are:	Eliminates possibility of extreme market opportunity loss under cost-plus pricing. Shares advantages of market price and cost-plus.	Shares disadvantages of market price and cost-plus.
(a) $P = k \cdot UCS$ if and only if $-M \le (MP - k \cdot UCS) \le M$; $MP \pm M$ otherwise.	(a) If cost-plus and market price differ by more than a constant M, the market price plus/minus this constant is used.	(a) Buyer and seller can stipulate a fixed maximum market opportunity loss.	(a) Price acquires variance of market price when "stuck" at the maximum price differential. May have lower or higher coefficient of variation than market price when "stuck".
(b) $P = k \cdot UCS$ if and only if $-m \cdot MP \le (MP - k \cdot UCS) \le m \cdot MP$; $MP \pm m \cdot MP$ otherwise.	(b) If cost-plus and market price differ by more than a percentage m of the latter, the market price plus/minus this percentage is used.	(b) Price acquires coefficient of variation of market price when "stuck" at the maximum price differential. May have lower or higher variance than price when "stuck".	(b) Buyer and seller cannot set a fixed maximum market opportunity loss. Buyer's and seller's protection from opportunity loss falls as opportunity loss rises.
(c) $P = k \cdot UCS$ if and only if $-m \cdot MP \le (MP - k \cdot UCS) \le m \cdot MP$; $MP - n(MP - k \cdot UCS)$ otherwise.	(c) If cost-plus and market price differ by more than a percentage m of the latter, the market price less a percentage n of this difference is used.	(c) The price does not "stick" at the maximum cost-plus, market price differential; consideration of costs are retained beyond this point.	(c) Neither fixed nor proportionate maximum opportunity loss is guaranteed.

(table continued)

TABLE 1.1 continued

Pricing formula[a/]	Description	Advantages[b/]	Disadvantages[b/]
Market price trend contact	Buyer pays seller a fraction the current period's market price MP_t plus a moving average of previous period's prices yet unpaid over the time horizon $t = 1, 2, \ldots, n$. There are variations in calculation of both current and residual components;	Eliminates possibility of lost market opportunity, on the average, over time horizon $t = 1, 2, \ldots, n$. Avoids fluctuations of market prices.	Requires a benevolent buyer who will handle periodic surpluses properly.
(a) $P = MP_t/X + \left(\frac{1}{n}\cdot\frac{X - 1}{X}\right)(MP_{t-1} + MP_{t-2} + \ldots + MP_{t-n})$	(a) The residual component's moving average weights each past year equally.	(a) Provides maximum price stability.	(a) Slow to react to new trend developments.
(b) $P = MP_t/X + 1/(a_1 + a_2 + \ldots + a_n)\left(\frac{X - 1}{X}\right)(a_1 MP_{t-1} + a_2 MP_{t-2} + \ldots + a_n MP_{t-n})$	(b) The residual component's moving average weights recent years greater than distant years.	(b) Prices become more responsive to recent market opportunities, new trend developments, as recent years' weights are increased.	(b) Prices become less stable as recent years' weights are increased.
(c) $P = \hat{MP}_t/X + \left(\frac{1}{n}\right)[(MP_{t-1} + MP_{t-2} + \ldots + MP_{t-n}) - (\hat{MP}_{t-1} + \hat{MP}_{t-2} + \ldots + \hat{MP}_{t-n})/X]$	(c) The current component may only be a preliminary estimate $\hat{MP}_t$ of the current market price; the residual component must then include compensatory payments where past estimates differed from realized prices.	(c) May be used when preliminary payments are sought before market price is known.	(c) Works well only when an adequate price prediction model is available.

(table continued)

TABLE 1.1 continued

Pricing formula[a/]	Description	Advantages[b/]	Disadvantages[b/]
Joint profit maximum[e/]	Buyer pays seller the price which maximizes the sum of profits of the two firms. The formula differs according to conditions in the external market:	Maximizes the sum of buyer and seller profit. Promotes cooperative behavior.	Assumes buyer and seller identify strongly with one another. Requires extensive data collection and analysis. May produce extensive price swings.
(a) $P = MRB - MRCB = MCS$	(a) If there is no external market, buyer pays seller that price equating seller's marginal cost MCS with buyer's marginal revenue net of marginal residual cost MRB - MRCB.		
(b) $P = MRB - MRCB = MCS - EMRS$	(b) If there is an imperfectly competitive external market, buyer pays seller that price equating seller's marginal cost net of external marginal revenue EMRS, with buyer's marginal revenue net of marginal residual cost.		
(c) $P = MP$	(c) If there is a perfectly competitive external market, buyer pays seller the market price; each trades freely on this market.		

a/ Some of the formulae listed under negotiated, market, cost-plus and sales-minus prices are taken from J. Dean and Cook. The inventory model as a market proxy is found in Chiang, p. 520. Profit share is derived from a common understanding of cooperative operation. Market price, cost-plus combinations are proposed by the authors. The market price trend contact is found in Bauer and Paish, and the joint profit maximum in Hirschleifer.

b/ Advantages and disadvantages apply to buyer, seller, or both, as indicated.

c/ Similar combinations may be designed for sales-minus and market price, or profit-share and market price.

d/ Differences between cost-plus and compromise prices may be paid at each delivery, or their sum may be payable at termination of contract.

e/ Marginal revenue is the increase or decrease in revenue caused by a unit increase in output. Marginal cost has a parallel definition. The formulae for a joint profit maximum are more exactly expressed in mathematical symbols, where ∂ indicates partial derivative and Q the buyer's output. In case (a) for example, the buyer sets $P = \partial RB/\partial Q - \partial CB/\partial Q$, and the seller sets $P = \partial CS/\partial Q$. Subtracting the latter from the former yields $\partial RB/\partial Q - \partial CB/\partial Q = \partial CS/\partial Q$, which maximizes the sum of the two firms' profits. In case (b) marginal revenues and marginal costs are similarly grouped to yield the classical profit maximum condition.

Profit share contracts are inspired by processing cooperatives' arrangements for compensating grower-members, but they need not be used only in a cooperative context. An essential requirement is that buyer and seller develop a strong common interest. A good example of this is profit share compensation for managerial labor.

The market price, cost-plus combination is proposed by the authors[1] to compensate for the possibility that cost-plus prices may fall too far out of line with external or "opportunity" market prices. Sales-minus and profit share contracts could similarly be specified with market price regulators.

A "market price trend contract" was proposed by Bauer and Paish with a view to standardizing the pricing policy of farmer marketing boards or stabilization schemes. The goal was to prevent such boards from accumulating excessive reserves or requiring external support, while simultaneously smoothing annual farmer income fluctuations.

The joint profit maximum price is owing to Hirschleifer, with later extensions by Gould and others. A buyer and seller employing this price maximize the sum of their profits if they severally maximize profit according to neoclassical economic theory. Such a scheme is best suited for goods transferred among a firm's decentralized profit centers.

1/ There is also reference to this idea in Williams.

II. STRUCTURE OF THE THEORETICAL AND EMPIRICAL MODELS

The objective of this report is to utilize the principles of decision making under risk in order to identify optimal sets of long-term contracts in a particular trading situation. The situation analyzed, initially developed in Buccola, includes a California processing cooperative, a customer for its bulk tomato paste, and a grower who supplies tomatoes to the coop on both a member and nonmember basis. The processor under investigation is a single pool, nontax-exempt cooperative that processes a number of fruits and vegetables other than tomatoes, including peaches, olives, and pineapple. The distributor who purchases the coop's tomato paste reprocesses it into tomato sauce. To simplify, the wholesale value of this sauce is assumed to be the distributor's only revenue source. The modeled representative grower raises corn, wheat, and dry beans in addition to processing tomatoes. All sales and purchase contracts between these firms stipulate fixed quantities and hold in force for ten years.1/ Cooperative membership through its board of directors is conceived as making the final coop marketing decisions in response to alternatives suggested by management. Processor returns are divided among members at the close of each market year. Hence, these returns measure revenue net of processing cost and nonmember raw product cost only; valuation of raw product delivered from members is excluded.

1/ The current usual practice in processing tomato markets is for grower and processor to sign a one-year fixed price contract before planting time in March. Processed tomato products are normally sold on a spot price basis. However, there is rising interest in developing suitable bases for long-term agreements in both raw and processed markets. Adaptation of one-year contracts to the model developed in this study is straightforward.

Decisions Under Risk

It is assumed that distributor, cooperative processor, and nonmember basis grower each seeks to employ a set of market contract formulae that maximizes its own expected utility, subject to the condition that terms will be mutually agreeable and trade will occur.

According to the concept of expected utility employed here, a firm's profit or net returns (π) are expressed as a function of: (1) a set of random revenue and cost terms (r, c), some of which represent alternative sales or purchase contract formulae for particular goods, and all of which are associated with specified probability distributions and covariance relationships; and (2) a set of decision variables (d) whereby the firm selects, from among the alternatives, a portfolio of marketing contracts through which to market its goods and purchase its resources. In general notation,

(1) $\pi = \pi(r, c; d)$.

Once a portfolio of contracts, and thus of revenue and cost terms r, c, are selected from among those specified in (1), the firm is thought to observe random values of these variables and consequently a random profit value.

Since profit is random, it is itself associated with a probability distribution that has, generally, mean, variance and higher moments. From formulae defining the moments of sums of random variables (J. Freund) it is clear, for example, that the mean (μ_π) and variance (σ^2_π) of (1) are functions of the means (μ_r, μ_c), variances (σ^2_r, σ^2_c), and covariances $\sigma_{r,c}$ of revenue and cost terms selected:

(2) $\mu_\pi = \mu_\pi(\mu_r, \mu_c; d)$

(3) $\sigma^2_\pi = \sigma^2_\pi(\sigma^2_r, \sigma^2_c, \sigma_{r,c}; d)$.

Each portfolio of market contracts considered is associated with an expected value of profit, and a variance of profit which represents the firm's risk.

The firm also possesses a preference ordering or utility function (U) through which it evaluates any profit figure proposed:

(4) $U = U(\pi)$.

Function (4) is assumed nondecreasing and continuous, and constitutes a cardinal measure in the limited sense that its uniqueness is preserved only under linear transformation. Utility function (4) expresses the firm's attitude toward risk. Over ranges for which its second derivative is negative, the variance of profit is weighted negatively and the firm is classified as risk averse. Where the second derivative is positive, profit variance is viewed positively and the firm is risk inclined. Linear functions indicate a concern only for expected values of net returns [Halter and Dean].

Since for each possible portfolio of contract revenue and cost terms, random profit drawings from (1) may be evaluated according to (4), each possible portfolio is associated with a probability distribution of utilities. The expected utility maxim identifies as the optimal portfolio that which produces the utility distribution with highest expected value. Selection of the maximum expected utility portfolio does not, however, require generation of utility probability distributions as such. If utility function (4) is given quadratic form, $U = a + b\pi - c\pi^2$, $b, c > 0$, expected utility may be expressed as

(5) $E[U(\pi)] = b\mu_\pi - c\mu_\pi^2 - c\sigma_\pi^2$,

where μ_π and σ_π^2 are defined as in (2), (3), and $E[U(\pi)]$ is expected utility.[1] Function (5) is quadratic and may be maximized by selection of decision variable values in (1), that is by choice of the appropriate marketing contract portfolio.

If the utility function is specified as negative exponential, $U = K - \theta \exp[-\lambda\pi]$, $K, \theta, \lambda > 0$, and if profit π is normally distributed, it may be shown [R. J. Freund] that expected utility is

$$(6) \qquad E[U(\pi)] = K - \theta \exp[-\lambda\mu_\pi + \lambda^2\sigma_\pi^2/2].$$

Function (6) is neither quadratic nor exponential, but the exponent is quadratic if profit function (1) is linear in the portfolio decision variables. Hence minimizing the exponent with respect to these decision variables effectively maximizes expected utility.

Many other forms of utility functions may be considered plausible, but most are difficult to estimate and work with empirically. The two forms described above are tractable for further analysis in that solutions may be obtained by ordinary quadratic programming methods.

Another, quite different approach seeks to avoid estimation of utility functions altogether by assuming (a) that the decision making firm is risk averse, and that (b) the distribution of profit or the decision maker's utility function is such that only mean and variance of profit need be considered in selecting marketing portfolios. An "efficient" set of portfolios is then derived from which the firm may select one best suited to its utility function. More particularly, if profit (1) is approximately normally distributed or utility (4) is quadratic, risk averters prefer, for any named expected value of profit, that portfolio of sales or purchase contracts which minimizes profit variance [Markowitz]. An efficient set of portfolios therefore consists of the minimum variance portfolio at each feasible level of expected profit. Variance minimization at each such point,

---

[1] In a more general approach, the expected utility function may be obtained from the Taylor Series expansion of U. For the first two moments, the expansion is $U(\pi) = U(\mu_\pi) + (\pi - \mu_\pi)U'\mu_\pi + (1/2)(\pi - \mu_\pi)^2U''(\mu_\pi)$. This has expected value $E[U(\pi)] = U(\mu_\pi) + (1/2)\sigma_\pi^2U''(\mu_\pi)$, a form equivalent to (5) upon substitution of the quadratic utility function.

$$(7) \qquad \text{Min } \sigma_{\pi}^{2}, \text{ s.t. } \mu_{\pi} = \mu_{\pi}^{\circ}, \text{ all feasible } \mu_{\pi},$$

constitutes a quadratic program if profit function (1) is linear in the decision variables.

Section four of this report utilizes the last method, referred to as "E-V analysis," in the investigation of the present market contracting problem. Section five presents estimates of representative utility functions for grower and cooperative processor, and section six employs some of these estimates to derive maximum expected utility marketing portfolios for the three modeled firms under assumptions of quadratic and negative exponential utility.

The Profit Functions

Profit functions are specified below which contain alternative sales or purchase contract instruments available to the modeled nonmember grower, cooperative and distributor. Each function also contains revenue and cost terms that are unavoidable and not subject to portfolio choice. Pricing arrangements considered for analysis are fixed price, market price, cost-plus, sales-minus, and profit-share.[1/] The first four are applicable to the distributor/reprocessor's purchase and cooperative's sale of bulk tomato paste; all five are applicable to the cooperative's purchase and

---

[1/] Fixed price arrangements are made by substituting a fixed price for the random market price.

growers' sale of raw tomatoes. These pricing formulae were chosen in response to discussions with cooperative leaders and growers; they appear to represent the class of contract options most appropriate to agricultural markets in the U.S. Limitations of research time prohibited inclusion of market proxy or the cost-plus, market price hybrid strategies (see Table 1.1). The joint profit maximum and market price trend contract options were not suitable to the institutional framework modeled.

The basic components of each profit or net margin function are the shares of transactions allocated to each contractual form (decision variables); random variables such as yields, market prices, and variable costs; and parameter values assigned to contractual terms. These functions are given in Tables 2.1 to 2.3. In each table, total annual profit or net revenue is the sum of the numbered terms in the table.[1/] The positive terms are revenue items and the negative terms are cost items. The symbols are defined in Table 2.4.

Defining the Cooperative Cost-Plus Sales and Sales-Minus Purchase Options

It will be observed that in the cooperative's costs of production to which the rate of return m is multiplied, the cost-of-raw-product portion is computed on the basis of the market price of tomatoes (cf. line 3), whereas the actual set of raw product costs may consist of any combination of market price, cost-plus, and sales-minus contracts. The reason for this simplification is that inclusion of actual cooperative raw product cost

---

[1/] The cooperative function is more properly described as a net margin function since it does not include on the cost side any valuation of the raw product transferred from member-growers. Throughout the balance of the report, cooperative net margins are usually referred to as such. Occasionally the word "profit" is used for convenience, for example in the term "profit share."

TABLE 2.1 Grower Profit Function

Component number	Component	Definitions
1	$+(OAAF)(REVF^{a}_{ntom})$	Revenue from all farm operations other than tomatoes.
2	$+S_1(AAF)(MP^{t}_{tom} \cdot Y^{a}_{tom})$	A proportion S_1 of revenue (earned from the allocated tomato acreage AAF) which is based on the per acre market value of tomatoes.
3	$+S_2(AAF)(k)(VCF^{a}_{tom})$	A proportion S_2 of revenue based upon the farmer's variable production cost times a rate of return k.
4	$+S_3(AAF)(\ell)(REVC)$	A proportion S_3 of revenue based upon total cooperative revenues (or cooperative paste revenues) times a markdown ℓ.
5	$+S_4(AAF)(z)(NMC)$	A proportion S_4 of revenue based upon a share z of cooperative net margin from all cooperative operations (membership contract).
6	$-(AAF)(VCF^{a}_{tom})$	Variable costs of tomato production.
7	$-(OAAF)(VCF^{a}_{ntom})$	Variable costs of nontomato (corn, wheat, bean) production.
8	$-FCF_{tom}$	Total fixed charges allocated to tomato production.
9	$-FCF_{ntom}$	Total fixed charges allocated to nontomato production.

TABLE 2.2 Cooperative Net Margin Function

Component number	Component	Definitions
1	$+REVC_{npst}$	Revenue earned from nonpaste processing operations.
2	$+V_1(AAC)(1/x)(MP^t_{pst} \cdot Y^a_{tom})$	A proportion V_1 of the market value of bulk tomato paste (market price sales option).
3	$+V_2(AAC)(m)(MP^t_{tom} \cdot Y^a_{tom})$	A proportion V_2 of the market value of raw tomatoes used in tomato paste manufacture, times the cooperative rate of return m.
4	$+V_2(AAC)(m)(1/x)(NTVCC^t_{pst} \cdot Y^a_{tom})$	A proportion V_2 of the nontomato cash cost of processing tomato paste, times the cooperative rate of return m. (Lines 3, 4 are the cost-plus sales option.)
5	$+V_3(AAC)(n)(1/xy)(MP^t_{sce} \cdot Y^a_{tom})$	A proportion V_3 of the distributor's revenue from tomato sauce (that was produced from the cooperative's bulk tomato paste) times the cooperative's revenue share n (sales-minus sales option).
6	$-R_1(AAC)(MP^t_{tom} \cdot Y^a_{tom})$	A proportion R_1 of the market value of tomatoes used in paste manufacture (market price purchase option).
7	$-R_2(AAC)(k)(VCF^a_{tom})$	A proportion R_2 of the total variable costs of farm production of processing tomatoes, times the farmer rate of return k (cost-plus purchase option).

(table continued)

TABLE 2.2 continued

Component number	Component	Definitions
8	$-R_3(AAC)(\ell)(REVC)$	A proportion R_3 of the cooperative's sales revenue from all operations, or all paste operations, times the farmer revenue share ℓ (sales-minus purchase option).
9	$-(AAC)(1/x)(NTVCC^t_{pst} \cdot Y^a_{tom})$	Total nontomato variable costs of bulk paste production.
10	$-VCC_{npst}$	Total variable costs allocated to all cooperative operations other than bulk paste.
11	$-FCC_{pst}$	Fixed costs allocated to all bulk paste production.
12	$-FCC_{npst}$	Fixed costs allocated to all cooperative operations other than bulk paste.

TABLE 2.3 Distributor Profit Function

Component number	Component	Definitions
1	$+(AAD)(1/xy)(MP^{t}_{sce} \cdot Y^{a}_{tom})$	Revenue earned from sale of tomato sauce at market prices.
2	$-W_1(AAD)(1/x)(MP^{t}_{pst} \cdot Y^{a}_{tom})$	A proportion W_1 of the market value of bulk paste used in sauce manufacture (market price option).
3	$-W_2(AAD)(m)(MP^{t}_{tom} \cdot Y^{a}_{tom})$	A proportion W_2 of the market value of raw tomatoes used in the manufacture of paste (which is in turn employed in sauce manufacture) times the coop's rate of return m.
4	$-W_2(AAD)(m)(1/x)(NTVCC^{t}_{pst} \cdot Y^{a}_{tom})$	A proportion W_2 of the non-tomato cash cost of processing bulk paste, times the coop's rate of return m. (Lines 3, 4 are the cost-plus option.)
5	$-W_3(AAD)(n)(1/xy)(MP^{t}_{sce} \cdot Y^{a}_{tom})$	A proportion W_3 of the distributor's revenue from tomato sauce sales, times the coop's revenue share n (sales-minus option).
6	$-(AAD)(1/xy)(NPVCD^{t}_{sce} \cdot Y^{a}_{tom})$	The distributor's nonpaste variable (costs of) processing tomato sauce.
7	$-FCD_{sce}$	The distributor's fixed (costs of) processing tomato sauce.

TABLE 2.4 Definitions of Terms for Tables 2.1 through 2.3

Terms	Definitions
(1) W_1, W_2, W_3	Nonrandom variables (proportions) by which the distributor chooses a portfolio of purchase contract options.
(2) V_1, V_2, V_3	Nonrandom variables (proportions) by which the cooperative chooses a portfolio of sales contract options.
(3) R_1, R_2, R_3	Nonrandom variables (proportions) by which the cooperative chooses a portfolio of purchase contract options.
(4) S_1, S_2, S_3, S_4	Nonrandom variables (proportions) by which the farmer chooses a portfolio of sales contract options.
(5) AAD, AAC, AAF	The acreage which, at expected yields per acre, the distributor (cooperative, farmer) calculates will be required to just meet target tomato sauce (tomato paste, tomato) production.
(6) Y^a_{tom}	Tomato yields in tons per acre.
(7) MP^t_{tom}	Market price of processing tomatoes at farm gate, in dollars per ton.
(8) MP^t_{pst}	Market price of tomato paste packed in 55-gallon containers, at paste plant, in dollars per ton.
(9) MP^t_{sce}	Market price of tomato sauce which is produced with bulk tomato paste as its principal input, at sauce plant, in dollars per ton.
(10) VCF^a_{tom}	Variable (cash) costs to produce an acre of processing tomatoes, Central Valley, California. (Additional o and ℓ subscripts represent owner and lessee costs, respectively.)
(11) $NTVCC^t_{pst}$	Nontomato variable (cash) costs to produce one ton of bulk tomato paste, including tomato transport to cannery.

(table continued)

TABLE 2.4 continued

Terms	Definitions
(12) $NPVCD^{t}_{sce}$	Nonpaste variable (cash) costs to produce one ton of tomato sauce, including paste transport to sauce plant.
(13) FCD_{sce}	Fixed costs incurred by the distributor for production of tomato sauce.
(14) FCC_{pst}	Fixed costs incurred by the cooperative for production of bulk tomato paste.
(15) FCF_{tom}	Fixed costs incurred by the farmer for production of processing tomatoes.
(16) $REVC_{npst}$	Revenue earned by the cooperative in its nonpaste processing operations.
(17) REVC	Revenue earned by the cooperative from all processing operations, or all paste processing operations.
(18) NMC	After-tax net margin earned by the cooperative from all processing operations, not including the market value of raw products transferred to the coop by farmer-members.
(19) $REVF^{a}_{ntom}$	Weighted average revenue per acre earned by the farmer from all nontomato operations.
(20) OAAF	Acreage allocated by the farmer for nontomato operations.
(21) VCC_{npst}	Variable (cash) costs allocated by the cooperative to its nonpaste processing operations.
(22) VCF^{a}_{ntom}	Weighted average variable (cash) costs per acre of the farmer's nontomato operations.
(23) FCC_{npst}	Fixed costs allocated by the cooperative to all nonpaste processing operations.

(table continued)

TABLE 2.4 continued

Terms	Definitions
(24) FCF_{ntom}	Fixed costs allocated by the farmer to all non-tomato operations.
(25) $m > 1$	The proportion of its cash costs charged by the cooperative to those with whom it has signed a cost-plus sales contract; "cost-plus markup."
(26) $n < 1$	The proportion of the sales revenue from tomato sauce which is returned to suppliers of bulk paste with whom the distributor has signed a sales-minus purchase contract; "sales-minus markdown." (1-n) is then the distributor's resale margin.
(27) $k > 1$	The proportion of its cash costs charged by the farmer to those with whom he has signed a cost-plus sales contract; "cost-plus markup."
(28) $\ell < 1$	The proportion of the sales revenue from all cooperative operations, or all paste operations, which is returned per acre to tomato farmers with whom the cooperative has signed a sales-minus purchase contract; "sales-minus markdown." $(1-\ell)$ is then the coop's resale margin.
(29) $z < 1$	The proportion of the after-tax net margin from all cooperative operations, not including the market value of raw products transferred to the cooperative by farmer-members, which is returned per acre to tomato farmer-members with whom the cooperative has signed a profit-share contract.
(30) y[a/]	Tons of tomato paste required to produce one ton of tomato sauce.
(31) x[a/]	Tons of raw processing tomatoes required to produce one ton of tomato paste.

[a/] Per ton variables MP^t_{pst}, $NTVCC^t_{pst}$, MP^t_{sce} are converted to a per ton raw product basis by appropriate conversions in x, y. In the remainder of this report, when per ton variables are expressed on a raw product basis they are signified with the superscript tr. The conversions are $MP^t_{tom} \equiv MP^{tr}_{tom}$, $MP^t_{pst}/x \equiv MP^{tr}_{pst}$, $NTVCC^t_{pst}/x \equiv NTVCC^{tr}_{pst}$, $MP^t_{sce}/xy \equiv MP^{tr}_{sce}$.

would involve a circular argument: one of these costs, signified by the proportion R_3, is computed on the basis of cooperative revenues less a resale charge $(1-\ell)$. But one of these latter revenue sources, signified by proportion V_2, is computed on the basis of cooperative costs. It is not possible for revenues and costs to be defined on the basis of one another, because the calculation of both awaits a proper definition of at least one of them. Thus, even though actual cooperative costs of buying raw product derive from some portfolio of market price, cost-plus, and sales-minus, the definition of production costs used in defining the V_2 revenue option (cost-plus) must be imputed to one of these alternatives, say market price, only.

A similar simplification is required to specify the cooperative sales-minus purchase (or grower sales-minus sales) option in the case where REVC represents paste rather than total revenue. If the modeled cooperative paid its sales-minus suppliers on the basis of its optimal contract revenue from tomato paste sales, the cooperative net margin function would be quadratic. That is, the sales-minus purchase option would contain sales portfolio proportions, V_i, $i = 1, 2, 3$ as well as the purchase proportion R_3. Since quadratic profit functions lead to quartic expected utility functions that are computationally intractable, cooperative paste revenue per ton is represented in option R_3 by the market price of paste. Thus, it is assumed for the purpose of this option that all cooperative paste is sold at market price.

The Expected Acreage Requirement

Use and construction of the expected acreage requirements are especially important in this model. Consider, for example, the coop's expected acreage requirement AAC. If the cooperative net margin were to be specified on the basis of the cooperative cannery's capacity in barrels of paste, then line 2 of Table 2.2, for example, would appear as $+ V_1(QP^\circ)(MP^t_{pst})$, where QP° is cannery capacity. This specification, however, does not accurately reflect a processing firm's planning behavior.

If sales of processed product are negotiated on a tonnage basis, the processor calculates the acres required, at current expected yields, to produce the requisite raw product, then seeks to contract this level of acreage. Expressing this in mathematical terms, $AAC = QP^\circ \cdot x/\bar{Y}^a_{tom}$, where $\bar{Y}^a_{tom}$ is expected tomato yields. Here cooperative revenues vary only with the per ton price of paste and do not depend upon tomato yields, although unexpected yield levels may inhibit contract compliance or swell processor inventories. In these circumstances the appropriate expression for paste market revenues is $MV^a_{tom} = (MP^t_{pst})(\frac{1}{x})(\bar{Y}^a_{tom})$.

If sales of processed product are negotiated on an acreage basis, the processor commits for sale the amount of product that may be processed in any year from a specified acreage AAC. In this case, revenues rise and fall with per acre yields as well as per ton prices; the per acre market value of paste is redefined as $\tilde{M}V^a_{pst} = (MP^t_{pst})(\frac{1}{x})(Y^a_{tom})$, where Y^a_{tom} is random.

The Future Profit Stream

The expressions in Tables 2.1, 2.2, and 2.3 outline the profit computations for a particular year, but long-term contract decisions require expected values (means) and variances of the discounted flow of profits over the life of the contract. In the empirical analysis this requires that we estimate the expected values and variances of each of the random price, cost, and yield variables listed in Tables 2.1, 2.2, and 2.3. The expected present value of the future profit stream may be obtained by replacing each random variable in the annual profit equation with its expected value, appropriately discounted for each future year of the contract, and summing over years as well as revenue and cost terms. The associated variance of the discounted profit sum is obtained by: (a) replacing each random variable in the profit equation with its variance, (b) squaring all nonrandom coefficients of these variables, including the appropriate discount terms, (c) introducing twice the covariance of each pair of random variables, retaining the coefficients of the original variables, and (d) summing over years as well as revenue and cost terms.

In the simple case where only one revenue term r and one cost term c are involved, the discounted profit sum π over T years is

$$\pi = \sum_{t=1}^{T} \frac{1}{(1+i)^t} (r_t - c_t) \tag{8}$$

where i is the annual interest rate. The associated expectation μ_π and variance σ_π^2 are then

$$\begin{aligned} E(\pi) &= \sum_{t=1}^{T} \frac{1}{(1+i)^t} [E(r_t) - E(c_t)] \\ \operatorname{var}(\pi) &= \sum_{t=1}^{T} \frac{1}{(1+i)^{2t}} [\operatorname{var}(r_t) + \operatorname{var}(c_t) - 2 \operatorname{cov}(r_t, c_t)]. \end{aligned} \tag{9}$$

Estimated values of ten-year discounted sums of means, variances, and covariances of random variables specified in Tables 2.1 to 2.3 are presented in the next section.

III. ESTIMATION OF EXPECTED VALUES, VARIANCES, AND COVARIANCES

In a purely static economy in which prices and costs varied randomly around fixed equilibrium values and the technology of production remained constant, the expected value of each variable could be estimated as the mean of historical values and the probability distributions could be estimated from the deviations around the means. In the real dynamic world, however, price, cost, and yield variables move over time and decision makers typically project such movements in forming their expectations as to future values of the variables. Thus, our estimates of expected values and variances must include a specification of the decision maker's projection system.

Projection Methods

Expected Values and Variances

In this study, we have assumed that the projections of the distributor, cooperative, and grower decision makers can be represented by simple linear trends. With this assumption, the present value of any cost or revenue variable X in a future year t can be identified by the equation

$$(10) \qquad X_t = (K + Bt + E_t)/(1 + i)^t$$

where K is the variable's current value, B a linear trend, E_t an error about the trend line, and i a discount rate. There are two sources of uncertainty regarding the value of X_t: the trend B which X will follow and the error E in any year t around the trend. Thus both B and E_t are random.[1/] The mean and variance of X_t are

---

[1/] Alternative developments of prediction probabilities are found in many sources, for example Johnston, pp. 152-155.

$$(11) \quad \begin{aligned} E(X_t) &= [K + tE(B) + E(E_t)]\frac{1}{(1+i)^t} \\ var(X_t) &= [t^2 var(B) + var(E_t) + (2t)cov(B, E_t)]\frac{1}{(1+i)^{2t}} . \end{aligned}$$

It is assumed, as in most regression analysis, that error expectation $E(E_t)$ is zero. Similarly, $cov(B, E_t)$ is zero under the reasonable presumption that the distribution of error terms does not change according to the trend selected. The simplified forms of (11) are

$$(11)' \quad \begin{aligned} E(X_t) &= [K + tE(B)]\frac{1}{(1+i)^t} \\ var(X_t) &= [t^2 var(B) + var(E_t)]\frac{1}{(1+i)^{2t}} . \end{aligned}$$

If discount rate $i = 0$, $E(X_t)$ rises or falls linearly with t and $var(X_t)$ rises quadratically with t. If $i > 0$, the possibilities for both functions are more complex. In particular, differentiating each function (11)' with respect to t indicates that discounted mean or variance may decline continuously over time; or either may rise, reach a maximum, and decline thereafter.

After estimates of $E(X_t)$, $var(X_t)$ are obtained, they are summed over the planning horizon $t = 1, 2, \ldots, T$. The result is a set of moments which reflect the probability of the present value sum of each variable X. In cases where the summed moments refer to price variables, summation is only meaningful if the associated input and output quantities are constants over the T years of the contract.

Covariances

Since profit is a function of price, cost, and yield variables, its variance is a function not only of the variances of these variables but also the covariances among them. In any year, the covariance between any two variables $X_{ht} = (K_h + B_{ht} + E_{ht})/(1 + i)^t$, $X_{jt} = (K_j + B_{jt} + E_{jt})/(1 + i)^t$ is

$$\text{(12)} \quad \mathrm{cov}(X_{ht}, X_{jt}) = [t^2\ \mathrm{cov}(B_h, B_j) + \mathrm{cov}(E_h, E_j)]/(1 + i)^{2t},$$

with correlation coefficient

$$\text{(13)} \quad \mathrm{cor}(X_{ht}, X_{jt}) = \frac{\mathrm{cov}(X_{ht}, X_{jt})}{\sqrt{\mathrm{var}(X_{ht})\ \mathrm{var}(X_{jt})}}$$

$$= \frac{t^2\ \mathrm{cov}(B_h, B_j) + \mathrm{cov}(E_h, E_j)}{\sqrt{[t^2\ \mathrm{var}(B_h) + \mathrm{var}(E_h)][t^2\ \mathrm{var}(B_j) + \mathrm{var}(E_j)]}}\ .$$

It is clear that (13), the correlation coefficient component of covariance (12), varies across time. Estimation of all correlations (13) would require estimates of the joint probabilities of linear trends B_h, B_j as well as of the covariances between linear residuals E_h, E_j.[1/] Owing to the difficulty of estimating $\mathrm{cov}(B_h, B_j)$, it is assumed here that correlation coefficients are invariant across time.

Correlations are estimated by inducing in each historical series X_h or X_j its expected future trend. This is accomplished by subtracting from each historical series the difference between its average annual historical change and predicted annual change E(B). The correlation matrix is then calculated in the usual way. If, for example, the variable X_h has an historical positive linear trend of 2 per year, and its mean predicted future trend is -1 per year, the difference in average annual change is -3 per year. Thus, zero is deducted from the initial value of X or X_1, -3 is deducted from X_2, -6 from X_3, -9 from X_4, and so on. Resulting correlation

---

[1/] The authors thank an anonymous reviewer for clarifying properties of the time paths of (11)', and for pointing out the relationships in (12) and (13).

coefficient estimates are finally multiplied by standard deviations of the included variables to form estimates of covariances of each pair of random variables specified in the profit functions, Tables 2.1 through 2.3.

These covariance estimates, together with summed expectation and variance estimates, now enable us to represent the present value sums of profits in Tables 2.1 to 2.3 which each decision maker faces over the planning horizon.

Converting Moments from a Tonnage to an Acreage Basis

It was explained in connection with profit function Tables 2.1 to 2.3 that expression of price and cost variables on an acreage rather than tonnage basis allows one to reflect the influence of yield risks on farmer, cooperative, and distributor decision making. If contracts specify sales on a tonnage basis, it is sufficient to specify yields as constant rather than random when constructing the data series. Thus, acreage-basis variables permit us to test both tonnage-basis and acreage-basis sales.

Several variables such as $REVF^{a}_{ntom}$, $VCF^{a,o}_{tom}$, $VCF^{a,\ell}_{tom}$, VCF^{a}_{ntom} are expressed on an acreage basis to begin with and so require no adjustment. Others such as $REVC_{npst}$, VCC_{npst} are totals with no fixed reference to tons or acres. However, the means and variances of MP^{tr}_{tom}, MP^{tr}_{pst}, MP^{tr}_{sce}, and $NTVCC^{tr}_{pst}$, were originally estimated on a tons raw tomato equivalent basis in order to simplify the work of isolating trend probability distributions p(B). They are converted to a per acre basis according to

$$(14)\quad E(X_iX_j) = \mu_i\mu_j$$

and

$$(15)\quad var(X_iX_j) = \mu_i^2\sigma_j^2 + \mu_j^2\sigma_i^2 + \sigma_i^2\sigma_j^2$$

where X_i refers to a price variable such as MP^{tr}_{pst} and X_j is tomato yield

Y^a_{tom}. Expressed on a per acre basis, MP^{tr}_{tom} becomes MV^a_{tom}, MP^{tr}_{pst} becomes MV^a_{pst}, MP^{tr}_{sce} becomes MV^a_{sce}, and $NTVCC^{tr}_{pst}$ becomes $NTVCC^a_{pst}$.

Expressions (14) and (15) are appropriate under the assumption that tomato yields are independent of annual tomato, paste, and sauce prices and annual paste production costs.[1/] This assumption seems reasonable since prices for raw tomatoes are set several months before per-acre yields are known, and paste and sauce prices are influenced closely by raw tomato prices and consumer demand. Moreover, yields are strongly dependent upon the exogenous weather factor.[2/] Covariances involving these variables are estimated by multiplying their standard deviations with correlation coefficients estimated from per-acre historical data.

Empirical Estimates

Estimates of the expected values (means), variances and covariances of each variable, following the above discussion, were developed in three steps. (1) Linear time trends were fitted by ordinary least squares to data series for the period 1951-1974. Decisions on long-term contract pricing alternatives were considered made in 1974 and the intercept values K were estimated as the 1974 trend value of the historical regression. The estimates of variance of the error term, $var(E_t)$, were obtained from the historical regressions. (2) Subjective projections and probability estimates of future linear trends B were developed from interviews with industry experts. (3) Expected future values and variances, $E(X_t)$ and $var(X_t)$, for

---

[1/] Proofs of these assertions, and an analysis of moments of product random variables, are found in Kmenta, pp. 57-66.

[2/] Such arguments were corroborated empirically. Regressions of tomato, paste, and sauce prices against Solano County yields gave insignificant results.

each variable were then obtained by substituting the estimates developed in steps (1) and (2) into equations (10) or (11)'. In the remainder of this section we will briefly describe the data series used in the analysis, present the historical variance estimates, describe and present the subjective trend estimates, and, finally, present the prediction means (expected values), variances and covariances of the profit variables.

Historical Data Series

The historical series for most variables, 1951-1974, are given in Table 3.1. All but yields are deflated by the wholesale price index (1974 = 100), since it is argued that decision makers in 1974 view historical price movements in terms of 1974 dollars. Similarly, price predictions for the future are expressed in 1974 dollars. Each variable series is explained briefly below. Sources and procedures involved in constructing the data series are described in Appendices C and D.

Yields (Y^{a}_{tom}). Tomato yields are an important factor in grower, cooperative, and distributor profit functions. Thus it is desirable to select historical yield series that best represent an individual grower's experience of yield variability on the one hand, and the more aggregate distributor's and cooperative's experience of yield variability on the other. However, collection of individual growers' yield records represents high cost in interview time; and county data, because it is aggregate, underestimates an individual farmer's yield variance. As an alternative, we have employed data from a "typical county" as a proxy for the experience of an individual grower. Solano County ($Y^{a,sol}_{tom}$) was selected for this purpose because it is an important county in tomato production and exhibits variances and coefficients-of-variation near the county weighted averages of these statistics.

TABLE 3.1 Annual Series of Revenue and Cost Variables Specified in Profit Functions of the Cooperative Processor and Growers, 1951-1974, 1974 Dollars[a/]

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Year	$Y^{a,sol}_{tom}$	$Y^{a,sc}_{tom}$	MP^{tr}_{sce}	MP^{tr}_{pst}	MP^{tr}_{tom}	$REVF^{a}_{ntom}$	$VCF^{a,o}_{tom}$	$VCF^{a,\ell}_{tom}$	$NTVCC^{tr}_{pst}$	$\tilde{MV}^{a}_{sce}$	$\tilde{MV}^{a}_{pst}$	$MV^{a,sol}_{tom}$	$MV^{a,sce}_{tom}$
			dollars						1974=100	dollars			
1951	18.68	17.46	454.92	119.63	53.07	129.93	562.39	647.43	61.80	7,942.92	2,088.75	991.46	926.60
1952	20.95	16.28	398.28	104.98	46.08	135.62	599.87	674.83	63.00	6,483.97	1,709.05	965.36	750.18
1953	18.68	19.54	425.11	100.52	41.95	136.36	558.51	609.95	65.80	8,306.67	1,964.10	783.61	819.70
1954	19.51	17.48	449.16	105.63	37.28	151.94	537.98	598.19	67.00	7,801.96	1,846.45	727.34	651.65
1955	20.20	16.67	495.84	127.97	41.58	170.73	549.82	618.27	69.50	8,255.72	2,133.26	839.82	693.13
1956	20.60	17.48	454.44	127.59	40.07	158.62	554.69	606.78	73.60	7,943.60	2,230.27	825.45	700.42
1957	16.12	16.16	413.14	102.35	37.58	158.77	552.35	564.72	78.30	6,676.46	1,653.96	605.74	607.29
1958	18.21	17.89	387.75	89.86	38.42	151.43	524.78	570.23	80.40	6,936.82	1,607.65	699.56	687.33
1959	14.92	14.51	391.94	92.17	36.82	153.82	568.18	567.86	83.40	5,687.06	1,337.34	549.33	534.25
1960	17.52	17.83	408.55	109.32	39.47	155.45	570.65	595.38	83.60	7,284.55	1,944.03	691.58	703.75
1961	14.15	15.73	422.67	127.72	50.99	161.31	579.94	615.96	85.20	6,648.43	2,009.09	721.53	802.07
1962	18.76	19.40	364.89	104.16	46.61	168.25	565.43	613.71	86.70	7,078.82	2,020.79	874.48	904.23
1963	19.26	20.31	384.45	107.91	43.03	173.64	582.52	622.04	89.10	7,808.22	2,191.68	828.73	873.93
1964	24.80	22.94	412.05	112.51	42.77	176.70	588.88	653.24	90.70	9,452.48	2,581.03	1,060.76	981.14
1965	20.34	20.95	450.65	145.48	58.67	185.76	672.72	745.61	93.80	9,441.07	3,047.85	1,193.30	1,229.13
1966	20.43	19.91	449.13	120.11	48.12	185.64	691.64	714.68	96.60	8,942.38	2,391.50	983.16	958.06
1967	17.77	17.69	480.69	157.08	61.96	178.13	672.94	737.30	100.00	8,503.46	2,778.77	1,101.02	1,096.07
1968	22.43	22.22	450.97	114.66	54.98	180.03	737.43	797.78	102.50	10,020.45	2,547.80	1,233.27	1,221.65
1969	22.35	22.39	445.96	80.83	40.89	166.81	760.82	751.34	105.30	9,985.06	1,809.85	913.89	915.52
1970	22.60	23.93	452.10	81.73	36.54	189.73	739.06	725.03	111.90	10,818.95	1,955.87	825.87	874.40
1971	24.41	23.30	447.62	90.34	39.36	183.90	708.28	770.19	119.80	10,429.59	2,104.86	960.75	917.08
1972	23.86	25.47	448.39	91.38	37.64	212.81	674.74	734.67	124.30	11,420.51	2,327.42	898.08	958.69
1973	23.57	22.50	428.47	90.83	41.60	361.50	772.52	807.42	133.50	9,640.47	2,043.69	980.57	936.00
1974	20.50	23.91	523.59	137.75	56.80	312.24	749.55	819.46	151.20	12,519.16	3,293.71	1,164.40	1,358.08
Correction factors[b/] (annual change)	--	--	--	--	--	+1.54	-10.15	-9.34	--	-163.07	-4.07	-5.95	--

[a/] Cooperative nonpaste revenue ($REVC_{npst}$), nonpaste cost (VCC_{npst}), and nontomato paste processing cost $NTVCC^{tr}_{pst}$ series have not been reported to preserve proprietary confidentiality. Interested persons should contact Tri/Valley Growers for access to this information.

[b/] These historical to predicted trend correction factors were calculated as explained in an earlier part of section IV.

The distributor's or cooperative's experience of yield variance ($Y^{a,sc}_{tom}$) is more nearly represented by average yields in the six counties from which the bulk of coop tomatoes is purchased. This average is weighted by the proportions of tomatoes purchased by the coop in each county in 1974. Hence, one would expect no important bias in the variance estimates from this series. Included counties with weights are San Joaquin, .25; Yolo, .20; Fresno, .17; Solano, .16; San Benito, .12; Stanislaus, .10.

U.S. Wholesale Prices of Consumer Size Tomato Sauce (MP^{tr}_{sce}). This variable is constructed to represent final tomato product prices received by the distributor/reprocessor, and is the basis for the sales-minus strategy for cooperative bulk paste sales. If the distributor's sales volume of all its final tomato products is a constant, its sales revenue from these products will vary in direct proportion to a weighted average of realized prices of these products, where the weights are the respective volume proportions which each product occupies in total sales.

Unfortunately, there is no specific information on the final product mixes of the modeled cooperative's principal customers for bulk paste. Since a large majority of the sales of a prospective contracting customer is occupied by tomato sauces, such as spaghetti and fish sauces, and the best available proxy for these are catsup prices, the latter are used to represent distributor's sales revenue.

U.S. Wholesale Prices of Bulk-Packed Tomato Paste (MP^{tr}_{pst}). Historical market price data for California tomato paste in various container sizes are available from a number of sources. Because the reported bulk-packed (chiefly 55-gallon) series begins in 1965 only, it was extended backward in time by reference to a correlated consumer-size series (see Appendix A).

Farm Prices for California Processing Tomatoes (MP^{tr}_{tom}). This series is reported at-farm because tomato transportation costs to cannery are considered a processing expense. It does not refer to a homogeneous product because the solids percentages of processing tomatoes oscillate widely across seasons and farms. The series represents only contract prices; open market prices are not easily obtainable and represent a small proportion of total tonnage in most years.

Farmer Nontomato Revenue Per Acre ($REVF^a_{ntom}$). The grower is here assumed to allocate his nontomato acreage to 50 percent corn, 30 percent wheat, and 20 percent dry edible beans. These were typical nontomato acreage proportions recorded in interviews with nine Central Valley tomato farmers. Per-acre revenue fluctuations for these commodities result from yield as well as price changes. The strong positive trend in this series is due largely to secular increases in corn and wheat yields.

Cooperative Nonpaste Revenue ($REVC_{npst}$). It is impractical to construct a weighted average revenue variable for $REVC_{npst}$, analogous to REV^a_{ntom} for the grower, due to the wide variety of products processed by the modeled cooperative. As an alternative, the cooperative has provided its aggregate annual revenue since 1964, the year of the firm's inception in its present form. Although used in subsequent statistical analysis, cooperative revenue and cost series are not reported in order to preserve confidentiality.

Farmer Historical Cash Cost of Tomato Production (VCF^a_{tom}). There is no published tomato production cost data from which the annual probability of these costs, as experienced by a typical tomato farmer, can be efficiently estimated. The best sources of production costs are the countywide studies performed by the University of California Cooperative Extension Service, which are put together by volunteer growers and updated irregularly. If there were available a 10- or 15-year series of annual updates for any county, one could employ this series to estimate the probability distribution of annual costs. Unfortunately, although annual figures are available, they refer to different counties and technological situations.

Construction of a cost series for a technologically constant firm proceeded in four phases. First, recent average per-acre input coefficients for such principal costs as labor, seed, fertilizer, pesticides, diesel, and water were estimated from California Agricultural Extension Service Reports on the six-county cooperative area. Second, a price series for these inputs was constructed covering 1951 to 1974. Third, the input coefficients were multiplied by corresponding unit input prices, 1951-1974, to obtain the total imputed per-acre costs of each input in each year. The latter were then summed across to give principal per-acre costs in each year, 1951-1974. Fourth, a miscellaneous cash cost factor, including office expenses, road maintenance, and land and equipment taxes, was added to principal costs to form the total cash cost series. An account of input coefficient and input price series construction for each principal cash input is provided in Appendix C.

Separate cost series are constructed for owner-operator ($VCF_{tom}^{a,o}$) and share-lessee ($VCF_{tom}^{a,\ell}$). As an accuracy check, our estimates of 1970-1974 total production costs in nondeflated form compare closely with published Extension Service cost totals for these years. For example, our 1973 nondeflated owner-operator total production costs are $769.58 per acre, compared to the 1973 Extension Service San Joaquin county estimate of $726.70. Our 1971 value is $623.53 per acre, compared with the 1971 Extension Service Solano county estimate of $626.30.[1/]

[1/] Owner-operated fixed cost of tomato production is assumed to be an unvarying $119.66 per acre for the purpose of this comparison (see Table 4.5).

The constructed series does not represent per-acre costs actually experienced in past years, since only recent technological coefficients are employed. This does not cause bias in subsequent variance estimates since cost-plus sales contracts frequently stipulate that cost savings due to technology change during the period of contract are inapplicable for computing transfer price. Thus, technology is defined at the time of contract signing and cost changes for pricing purposes result from price changes only.[1/] This is known as a "no pass through" provision.

Farmer Historical Cash Cost of Corn, Wheat, and Bean Production (VCF^a_{ntom}). Corn, wheat, and bean cost-of-production studies on a county-wide basis are also published by the University of California Cooperative Extension Service. It is possible to construct time series of cash costs for these crops in an analogous manner to tomato costs. Since these costs do not enter directly into the decision making process in the expected utility programs, however, it would not be a good use of research time to treat them as we have tomato costs. An alternative taken here is to assume that the ratio of standard deviation to mean of costs for each of these crops is equal to that of tomato costs. The mean of corn, wheat, and bean costs can be taken from current studies and the variances correspondingly derived. The variance formula is

$$(16) \qquad \frac{\mathrm{var}(VCF^a_{ntom})}{[E(VCF^a_{ntom})]^2} = \frac{\mathrm{var}(VCF^a_{tom})}{[E(VCF^a_{tom})]^2} .$$

[1/] The implied assumption is that 1951-1974 factor price variance is a good estimate of 1975-1984 factor price variance. Suppose, however, that the 1974 technology was available in 1951 but not adopted due to high factor prices (e.g., color television or transistors). The early 1950's price series would be abnormally high and would positively bias the variance of these prices. Labor, fuel, and electricity for tomato production would not appear to fall under such a supposition.

Cooperative Historical Nontomato Cash Cost of Tomato Paste Production ($NTVCC^{tr}_{pst}$). This series was estimated in a manner similar to the farmer cost series, VCF^{a}_{tom}. It applies the "no-pass-through" provision and employs the set of input coefficients prevailing in 1974, the year a long-term contract is assumed to have been signed. Thus, cost variations result from input price changes only and do not reflect potential technology improvements.

The cooperative under study operates four processing plants that handle tomato products; three of these process tomato paste that is packed in 55-gallon barrels and sold to distributors for reprocessing. The ideal production cost series would reflect the average cash cost to produce a barrel of paste at these three plants, weighted by the proportions of barrel paste produced at each plant. Unfortunately, the production conversion coefficients are available for one plant only. Since this plant is more capital intensive than the others, and capital costs are measured as fixed as opposed to cash, the cash cost series is probably biased downward.

The major components of bulk paste packing costs are labor, steel drum, electricity, gas, tomato and refuse hauling, and the tomatoes themselves. The proportion of tomato cost to total cost is in the area of 60 to 70 percent, although this varies radically with annual changes in the market price of tomatoes. Since the tomato market price is handled separately under MP^{tr}_{tom}, only nontomato costs are dealt with here.

The historical series of total principal cash costs is formed by multiplying the per-barrel input coefficient of each principal input (labor, barrel, power, hauling) by its corresponding annual input price, and adding these products across each year. To principal costs are added a miscellaneous cost factor, formed by taking the ratio of miscellaneous to principal costs in a recent cooperative cost study and multiplying this by the principal cost series. The sum of principal and miscellaneous costs constitutes total nontomato cash cost. This is an imputed series, meaning that it is an estimate of costs the cannery would have incurred had it operated from the period 1951-1974. An index form of this series is shown in Table 3.1 to protect source confidentiality. Appendix C contains an account of price series construction for each principal nontomato cash input.

Cooperative Historical Cash Costs Allocated to Production of Items Other than Bulk Paste (VCC_{npst}). An annual series of total cooperative cash costs, covering direct production and selling expenses, was computed for 1965-1974 from data supplied by the cooperative under investigation. From this series was deducted a portion of costs allocated to drum paste production.

Value-Per-Acre Variables (MV^{a}_{sce}, MV^{a}_{pst}, $MV^{a,sc}_{tom}$, $MV^{a,sol}_{tom}$). The first three of these are calculated by multiplying the appropriate per-ton raw equivalent price variable by six-county yields ($Y^{a,sc}_{tom}$) and so are appropriate for use in distributor and cooperative models. The last is derived from Solano county yields ($Y^{a,sol}_{tom}$) instead, and hence appropriate for the grower model. Inasmuch as MV^{a}_{pst} is a six-county variable, it is also a good basis for the grower sales-minus purchase option in Table 2.3.

Historical Statistical Analyses and Estimates of K, var(E_t)

Results of linear trend fits to the deflated historical series are presented in Table 3.2. All trends are statistically significant at the 95 percent level except for tomato, paste, and sauce market prices. In the latter cases, residual variances recorded refer to the nontrended series. Average annual historical changes for each variable are recorded

TABLE 3.2

Summary of Results from Time Trend Fits
to Deflated Revenue and Cost Variables

Deflated variables	Linear trend		
	Residual variance $[var(E_t)]$	$\bar{R}^2$	Annual historical change a/
		(dollars)	
(1) $REVF^{a}_{ntom}$	1,371.220	.497	5.310 (4.870)
(2) MP^{tr}_{tom}	57.460	-.013	.187 (.832)
(3) $VCF^{a,o}_{tom}$	1,748.910	.744	10.150 (8.230)
(4) $VCF^{a,\ell}_{tom}$	2,488.010	.631	9.340 (6.350)
(5) $REVC_{npst}$	132.710 b/	.697	c/
(6) VCC_{npst}	126.180 b/	.532	c/
(7) MP^{tr}_{pst}	396.410	-.038	-.235 (-.392)
(8) $MTVCC^{tr}_{pst}$	.667	.870	.300 (12.450)
(9) MP^{tr}_{sce}	1,271.640	.056	1.610 (1.530)
(10) $Y^{a,sol}_{tom}$	6.074	.246	.212 (2.920)
(11) $Y^{a,sc}_{tom}$	3.504	.630	.350 (6.330)

a/ t-values are given in parentheses.

b/ Values are in trillions of dollars.

c/ These figures have not been reported in order to preserve proprietary confidentiality. Interested persons should contact Tri/Valley Growers for access to this information.

for the reader's interest but have no immediate bearing on the distribution of future trends B.

Estimates of Subjective Trend Probabilities p(B)

The probability distributions of B employed to calculate means and variances in (11)' are calculated from subjective projections of industry and academic experts. A body of literature has developed on the subject of subjective probability elicitation; the literature emphasizes techniques that promote honesty and systematic introspection in respondents [Hampton, *et al*.]. In the present study, advantage was taken of inflation prospects to ensure that responses were careful. An expert was first asked to name the most likely value, in 1974 dollars, that the variable in question would have in 1984. Then he was asked what probability he assigned to observing a value in 1984 some percentage above or below this most likely value. To corroborate these responses, and to ensure the individual was thinking in 1974 dollars, he was next asked to consider whether the variable in question would rise faster, slower, or at the same rate as inflation, whatever the inflation rate may be. Probabilities were elicited for the prospects of the variable rising at selected rates above or below the future rate of inflation. If responses to the second approach were not consistent with those to the first, each approach was repeated until consistency was achieved.

If in the respondent's final judgment, a variable was expected to move with inflation exactly, its 1984 expected value was recorded equal to its 1974 value calculated at the linear trend line. This condition held true for VCF^{a}_{tom} and $NTVCC^{tr}_{pst}$. Variables $REVF^{a}_{ntom}$, $REVC_{npst}$, VCC_{npst} had expected 1984 projection values higher than their 1974 linear trend values K. For variables MP^{tr}_{tom}, MP^{tr}_{pst}, MP^{tr}_{sce}, and Y^{a}_{tom}, the 1984 mean projections were lower than their 1974 linear trend values.

Results of all trend probability projections are recorded in Table 3.3. Column (1) records each variable value on its respective historical trend line in 1974. Column (4) lists respondents' subjective probabilities of the 1984 trend values recorded in column (2). Column (3) identifies annual average changes in the variable implied by each 1984 trend value named. Means, E(B), and variances, var(B), calculated from columns (3) and (4) are shown in columns (5) and (6).

The probability distributions of linear trends shown in columns (3) and (4) are not aggregates but represent responses of individuals. The tomato yield distribution was elicited from a plant scientist. Farm production cost distributions were obtained from a tomato grower; several subsequent growers evinced cost trend predictions not very different from those shown here. All other probability distributions were obtained from a spokesman for the modeled cooperative. Most probability respondents were principally guided by their general sense of bullishness or bearishness over future industry profits. The profit prospects of all individuals were in fact bullish with regard to both farmer and processor tomato profits. The general feeling was that 1974 farmer and processor tomato profits were excessive and atypical, but that profits in the next ten years would average better, in real terms, than the past ten years.

Calculation of Ten-Year Probability Moments

Annual discounted means, $E(X_t)$, and variances, $var(X_t)$, of all variables can readily be calculated by substituting into formulae (11)' the values for E(B), var(B), K, and $var(E_t)$ shown in Tables 3.2 and 3.3. Since we are dealing with deflated prices, it is appropriate to use a

TABLE 3.3

Calculation Procedures to Obtain Estimates of Probability Moments of Future Linear Trends (B), Revenue and Cost Variables, 1975-1984

	(1) 1974 trend value (K)	(2) 1984 predicted trend values	(3) Annual predicted change	(4) Indicated probabilities[a/]	(5) E(B)	(6) var(B)
Yields (Y^{a}_{tom}) (tons per acre)	23.68[b/]	26.50	.282	.4	.492	.0369
	23.68	29.50	.582	.5		
	23.68	32.50	.882	.1		
Tomato market price (MP^{tr}_{tom}) (dollars per ton)	44.60	32.50	-1.210	.1	-.460	.2625
	44.60	37.50	-.710	.6		
	44.60	47.50	.290	.3		
Tomato cost of production, owner[c/] ($VCF^{a,o}_{tom}$) (dollars per acre)	780.83	741.79	-3.904	.2	0	6.0965
	780.83	780.83	0	.6		
	780.83	819.87	3.904	.2		
Paste market price (MP^{tr}_{pst}) (dollars per ton raw equivalent)	110.11	85.37	-2.474	.1	-.699	1.5251
	110.11	97.01	-1.310	.6		
	110.11	121.25	1.114	.3		
Bulk paste processing cost ($NTVCC^{tr}_{pst}$) (dollars per ton raw equivalent)	[d/]	[d/]	-.113	.2	0	.0051
			0	.6		
			.133	.2		
Spaghetti sauce market price (MP^{tr}_{sce}) (dollars per ton raw equivalent)	453.48	324.48	-12.900	.1	-6.149	22.0400
	453.48	368.73	-8.470	.6		
	453.48	460.91	.743	.3		
Other farmer revenue ($REVF^{a}_{ntom}$) (dollars per acre)	241.92	257.01	1.509	.1	6.854	13.8350
	241.92	292.06	5.014	.6		
	241.92	365.07	12.315	.3		
Coop nonpaste cash costs (VCC_{npst})	[d/]	[d/]	1.685	.2	2.077	.0642
			2.072	.6		
			2.486	.2		
Coop nonpaste revenue ($REVC_{npst}$)	[d/]	[d/]	2.157	.2	2.692	.1153
			2.692	.6		
			3.230	.2		

[a/] These probabilities are derived from interviews with industry representatives.

[b/] Only the six-county intercept is used since six-county rather than Solano expected yields are utilized in moment calculations.

[c/] Tomato production cost under land lease, $VCF^{a,\ell}_{tom}$, was assumed to have the same linear trend (B) distribution as owner costs $VCF^{a,o}_{tom}$. Only residual variances were allowed to differ as shown in Table 3.2.

[d/] Value omitted to preserve confidentiality of private data supplied by the cooperative.

discount rate that approximates the expected real rate of growth in the U.S. economy during the forecast period. In this study the rate $i = .02$ is utilized, which reflects some pessimism over the real economic growth rate but is not unduly disastrous.

Rather than employ the above indicated analytical method of generating annual means and variances, we have employed a simulation program in which values of K and $\mathrm{var}(E_t)$, and discrete distributions of B, were substituted into projection formula (10). Three hundred sample values of each random price, cost, and yield variable were then drawn for each of the ten years of prediction, and estimates of $E(X_t)$, $\mathrm{var}(X_t)$ derived from the sample values. This procedure introduces some sampling error to the estimation process. However, these errors are not great under the sample size drawn. The simulations provided data from which Chi-square tests were performed on hypotheses of probability functional forms. Tests of the hypothesis that each data set was drawn from a normal distribution were performed for each simulated 1979 data set [J. Freund, pp. 337-338]. This hypothesis was not rejected for any variable at the five percent level of significance. The result is somewhat surprising since, although trend deviations E_t were assumed normally distributed, some trend distributions B were skewed.[1/]

Ten-year sums of discounted annual means and variances of the price, revenue, and cost variables are listed in Table 3.4. Moments listed for yields are ten-year averages, rather than sums, to facilitate multiplication with price variables. All moments of revenues and costs are then expressed on a per-acre basis in Table 3.5. Per-acre moments were calculated from

---

[1/] Since the nonnormal variables (B) are weighted most heavily in 1984, Chi-square tests in that year would have provided even more stringent tests of overall normalcy.

TABLE 3.4

Estimates of Ten-Year Sums of Discounted Means and Variances: Yields, Revenue and Cost Variables

Variable	Sum of discounted means $[\sum_t E(X_t)]$	Sum of discounted variances $[\sum_t \mathrm{var}(X_t)]$
$Y^{a,sc}_{tom}$	26.347[a]	.489[a]
$Y^{a,sol}_{tom}$	26.326[a]	.739[a]
$REVF^{a}_{ntom}$	\$2,501.03	\$15,224.60
MP^{tr}_{tom}	\$378.95	\$548.10
$VCF^{a,o}_{tom}$	\$7,018.18	\$16,075.53
$VCF^{a,\ell}_{tom}$	\$7,019.70	\$22,160.16
VCC_{npst}	[b]	\$1,059.77[c]
$REVC_{npst}$	[b]	\$1,125.62[c]
MP^{tr}_{pst}	\$956.80	\$3,700.46
MP^{tr}_{sce}	\$3,775.78	\$16,745.54
$NTVCC^{tr}_{pst}$	[b]	6.912

[a] Values for $Y^{a,sc}_{tom}$ and $Y^{a,sol}_{tom}$ are ten-year averages.

[b] Expected values for cooperative total revenue, total cost, and paste processing costs have not been reported to preserve confidentiality.

[c] Values are in trillions of dollars.

TABLE 3.5 Means and Variances of Present Value Sums of Profit Variables Expressed on a Per-Acre Basis

Variable	Mean	Variance of product
	dollars per acre	
$REVF^{a}_{ntom}$	2,500.5	15,224.6
$MV^{a,sol}_{tom}$	9,982.88	487,043.3
$MV^{a,sc}_{tom}$	9,982.88	450,986.15
$VCF^{a,o}_{tom}$	7,018.2	16,075.5
$VCF^{a,\ell}_{tom}$	7,018.2	22,160.2
VCF^{a}_{ntom}	1,977.3	1,759.0
$REVC_{npst}$	a/	1,125.6[c/]
REVC	a/	1,107.0[c/]
$\tilde{MV}^{a}_{pst}$	25,208.81	3,018,503.348
MV^{a}_{pst}	25,208.81	2,568,755.36
$\tilde{MV}^{a}_{sce}$	99,481.00	18,608,110.2
MV^{a}_{sce}	99,481.00	11,624,129.96
$NTVCC^{a}_{pst}$	a/	25,062.92
VCC_{npst}	a/	1,059.8[c/]
$FCF^{a,o}_{tom}$	1,196.6	--
$FCF^{a,\ell}_{tom}$	1,136.6	--
FCF^{a}_{ntom}	363.0	--
FCC_{npst}	a/	--
FCC^{a}_{pst}	a/	--
$NPVCD^{a}_{sce}$	35,665.93	1,144,852.7[b/]
FCD^{a}_{sce}	15,648.80	--

a/ These figures have not been reported in order to preserve proprietary confidentiality. Interested persons should contact Tri/Valley Growers for access to this information.

b/ Calculated from the ratio of variance to mean square of $NTVCC^{a}_{pst}$; that is $var(NPVCD^{a}_{sce})/[E(NPVCD^{a}_{sce})]^2 = var(NTVCC^{a}_{pst})/[E(NTVCC^{a}_{pst})]^2$.

c/ Values are in trillions of dollars.

per-ton moments according to equations (14), (15). Note that $\tilde{M}V^{a}_{pst}$ and $\tilde{M}V^{a}_{sce}$ differ from MV^{a}_{pst} and MV^{a}_{sce} in that the former are computed on the basis of random yields and the latter on the basis of a constant yield. As discussed above, those constructed under random yields are suitable for modeling acreage-basis contracts, whereas those constructed under constant yields are suitable for tonnage-basis contracts.

Covariances among all revenue, price, and cost variables are given in Table 3.6. Standard deviations σ_1, σ_2 are square roots of prediction variances as reported in Table 3.5. Correlation coefficients r_{ij} are computed from the historical series given in Table 3.1, as adjusted by the trend correction factors listed at the bottom of that table.

It will be noticed that correlations involving MV^{a}_{pst}, MV^{a}_{sce} are algebraically lower than corresponding correlations for which $\tilde{M}V^{a}_{pst}$, $\tilde{M}V^{a}_{sce}$ are substituted. For example, $cor(NTVCC^{a}_{pst}, MV^{a}_{pst}) = -.297 < cor(NTVCC^{a}_{pst}, \tilde{M}V^{a}_{pst}) = .429$. This is because the latter pair share a common random yield component and the former do not. In fact, many of the rather strong positive correlations observed are due to both variables containing the same random yield factor Y^{a}_{tom}.

TABLE 3.6 Prediction Correlation Coefficients and Covariances Among Revenue and Cost Variables a/

	$REVF^{a}_{ntom}$	$MV^{a,sol}_{tom}$	$MV^{a,sc}_{tom}$	$VCF^{a,o}_{tom}$	$VCF^{a,\ell}_{tom}$	$REVC_{npst}$	VCC_{npst}	$\tilde{MV}^{a}_{pst}$	MV^{a}_{pst}	$\tilde{MV}^{a}_{sce}$	MV^{a}_{sce}	VCF^{a}_{ntom}	$NTVCC^{a}_{pst}$	$NPVCD^{a}_{sce}$
$REVF^{a}_{ntom}$	--													
$MV^{a,sol}_{tom}$	22.646 (.263)	--												
$MV^{a,sc}_{tom}$			--											
$VCF^{a,o}_{tom}$	1.079 (.069)	40.523 (.458)		--										
$VCF^{a,\ell}_{tom}$	2.461 (.134)	80.616 (.776)			--									
$REVC_{npst}$			.81086* (.036)	.29765* (.070)	.98857* (.198)	--								
VCC_{npst}			.04372* (.002)	1.0193* (.247)	.90124* (.186)	1,037.14** (.950)	--							
$\tilde{MV}^{a}_{pst}$			917.063 (.786)	10.793 (.049)	101.640 (.393)	2.1560* (.037)	-3.1669 (-.056)	--						
MV^{a}_{pst}			553.229 (.514)	-1.219 (-.006)	69.427 (.291)	-13.3851* (-.249)	-18.3113* (-.351)		--					
$\tilde{MV}^{a}_{sce}$			1,489.00 (.524)	197.974 (.362)	353.176 (.550)	54.2556* (.375)	65.1507* (.464)	3,829.708 (.511)		--				
MV^{a}_{sce}			50.371 (.022)	104.171 (.241)	165.453 (.326)	-45.2831* (-.396)	-39.8404* (-.359)		2,431.635 (.445)		--			
VCF^{a}_{ntom}		13.405 (.458) b/		5.317 (1.00) c/	6.243 (1.00) c/			797.509 (.429) d/				--		
$NTVCC^{a}_{pst}$			36.360 (.342)	-.080 (-.004)	1.178 (.050)	1.6620* (.313)	1.9582* (.380)	117.996 (.429)	-75.358 (-.297)	282.725 (.414)	-395.096 (-.732)		--	
$NPVCD^{a}_{sce}$			245.749 (.342) e/					797.509 (.429) e/	-509.332 (-.297) e/	1,910.883 (.414) e/	-2,670.378 (-.732) e/		-677.58 (-.004) f/	

a/ In each cell, covariances are listed above and correlation coefficients listed below in parentheses. Covariances are expressed in thousands of dollars, except for those asterisked. Those with single asterisks are expressed in billions of dollars and with double asterisks in trillions of dollars.

b/ This correlation is taken from cor($VCF^{a,o}_{tom}$, $MV^{a,sol}_{tom}$) due to lack of data series for VCF^{a}_{ntom}.

c/ The perfect correlations were assumed. There was no data series for VCF^{a}_{ntom} or $NPVCD^{a}_{sce}$ in order to determine them empirically.

d/ This correlation is taken from cor($NTVCC^{a}_{pst}$, $\tilde{MV}^{a}_{pst}$) due to lack of data series for VCF^{a}_{ntom}.

e/ Each of these correlation coefficients is taken from that where $NTVCC^{a}_{pst}$ is substituted for $NPVCD^{a}_{sce}$. For example, cor($NPVCD^{a}_{sce}$, $\tilde{MV}^{a}_{sce}$) = cor($NTVCC^{a}_{pst}$, $\tilde{MV}^{a}_{sce}$). There were no data to estimate these empirically since a series for $NPVCD^{a}_{sce}$ could not, in the absence of a cooperating distributor, be developed.

f/ This correlation is taken from cor($NTVCC^{a}_{pst}$, $VCF^{a,o}_{tom}$) due to lack of data series for $NPVCD^{a}_{pst}$.

IV. EFFICIENT PORTFOLIOS OF CONTRACTUAL ARRANGEMENTS

The theoretical framework outlined in Section II showed that the utility of any set of contractual arrangements may be expressed as a function of the utility of the expected monetary outcome and the variance of these returns. The portfolio choice considered optimal varies among individuals according to their utility functions for money. We shall explore the optimal choice space using representative utility functions for various groups of participants. Before doing so, it is important to note that many contract portfolios can be eliminated as inefficient for any risk averse decision maker. Intermediate solutions which identify efficient portfolios thus may be valuable regardless of any knowledge of the specific forms of utility functions. In this section we present such solutions and their associated E-V curves for each of the contract participants--growers, cooperative, and distributor--for selected sets of alternative contract specifications.

Efficient portfolios are determined, as indicated in Section II, by solving quadratic programming problems which minimize variance, σ_π^2, for given expected profit levels, μ_π. Expected profits are measured as the discounted sum of expected annual profits over the ten-year contract horizon. Expected annual profits are computed in accordance with Tables 2.1 to 2.3 in Section II, with the price and cost variables set at their expected values, as calculated in Section III. With specific values assigned to the acreage variables and contract parameters, expected profit may be expressed as a linear function of the shares allocated to each contract option. Variances of profit, computed as indicated in equation (9), Section II, similarly may be expressed as (quadratic) functions of shares assigned to each contract option. A listing of profit expectation and variance expressions used in E-V analyses of grower, cooperative, and distributor operations is given in Appendix A. Points on the E-V curve or "efficiency frontier" are determined by parametrically altering expected profit levels. E-V solutions were obtained by a quadratic programming routine developed by Haegert and Harris.

Grower Efficiency Frontiers

The grower model examined here sets the cost-plus markup (k) at 1.25, the sales minus markdown (ℓ) at .385 and the grower's share (z) of cooperative net margin at .000029. The latter is the 1974 average proportion of coop net margin received by tomato grower members for each acre of tomatoes delivered to the cooperative under study. The grower is assumed to have devoted 800 acres to corn, wheat, and bean production. Three situations are considered with respect to total tomato acreage and land tenure arrangements: (1) 500 acres in tomatoes on leased land, (2) 500 acres in tomatoes on own land, and (3) 1,000 acres in tomatoes on own land.

The grower may sell to the cooperative on either or both a member or a nonmember basis. Thus, the portfolio possibility set includes membership or profit share as one of the options. The farmer's preference for this option depends upon his net margin share z (in this case, the amount of the farmer's membership sales as a proportion of total cooperative membership purchases) and the mean and variance of coop net margin that is assumed. Therefore, each farmer E-V frontier, especially as long as it includes the patronage option S_4 as a variable, assumes not only a specific cooperative E-V frontier but a particular solution point on this frontier. Even if S_4 were to be constrained at or above some value, the levels of other proportions S_1, S_2, S_3 would be affected by the cooperative net margins assumed. This would be due to the presence of correlations between these net margins and other farmer profit variables such as the tomato market price. It is assumed in the frontiers included here that the expected ten-year present value sum of annual coop net margins is \$355 million and the ten-year present value sum of variances is \$349.8 trillion.[1]

Efficient portfolio solutions for the three acreage-tenure variations are given in Table 4.1.[2] The associated E-V curves are graphed in Figure 4.1. The diagrams show the trade-off or efficiency frontiers between expected value of profit (μ_π) and coefficient of variation of profit (σ_π/μ_π) as well as between expected value and variance of profit. All figures refer to ten-year sums of discounted grower profits.

Grower E-V curve #1, Figure 4.1A, is "classically" shaped in the sense that it rises positively and approximately quadratically throughout its range. However, the associated coefficient-of-variation curve is negative over much of its range, and is more erratically shaped. The negative range of this curve reflects the very gradual positive slope of the E-V frontier over the expected profit range .437 to .830. It would be an extremely risk averse grower indeed who would operate below the .830 level, where the efficient portfolio is S_1 = 21.6 percent, S_2 = 43.8 percent, S_3 = 0, S_4 = 34.5 percent.

Efficiency set #1 assumes that the model farmer leases his farm land on a share-rent basis. If the farmer owns his land instead, as in E-V #2, Figure 4.1B, the variance of his production costs decreases (see Table 3.5). We would expect the E-V curve for owner-operators to shift downward,

---

[1] This may be noted as point A on the cooperative efficiency frontier #3, Figure 4.2C, discussed subsequently. Note that although these variances are actually denominated in "squared dollars," dollar signs are used for the sake of simplicity.

[2] The optimizing model is described in Appendix A.

TABLE 4.1 E-V and Coefficient of Variation Frontiers Indicating Efficient Portfolios of Market Price, Cost-Plus, Sales-Minus, and Cooperative Member Sales Contracts for a California Producer of Processing Tomatoes

Efficiency set number	Moments and coefficients of profit				Tomato sales[a]				Number of iterations[b]
	Expected profit[c]	Variance[c]	Standard deviation[c]	Coefficient of variation	Market price (S_1)	Cost-plus (S_2)	Sales-minus (S_3)	Coop member (S_4)	
	million dollars	billion dollars	million dollars		proportions of portfolio				
1	.437	11.000	.10488	.2400	0	1.000	0	0	1
	.570	11.747	.10838	.1901	0	.766	.122	.101	7
	.670	14.479	.12033	.1796	.098	.656	.040	.206	9
	.830	18.886	.13743	.1656	.216	.438	0	.345	7
	.950	34.050	.18453	.1943	.198	.240	.114	.447	9
	1.075	46.125	.21477	.1998	.358	.088	0	.554	9
	1.110	50.120	.22387	.2017	.401	0	.085	.514	9
	1.198	102.000	.31937	.2666	0	0	0	1.000	1
2	.437	10.000	.10000	.2288	0	1.000	0	0	1
	.570	10.953	.10466	.1836	.050	.823	0	.127	4
	.670	12.775	.11303	.1687	.135	.616	.129	.119	4
	.830	28.832	.16980	.2046	.128	.424	.085	.363	10
	.950	41.375	.20341	.2141	.202	.242	.110	.446	10
	1.075	57.432	.23965	.2229	.322	.096	0	.582	8
	1.110	63.130	.25126	.2264	.333	.047	0	.619	8
	1.198	102.000	.31937	.2666	0	0	0	1.000	1
3	.746	10.000	.10000	.1340	0	1.000	0	0	1
	1.000	17.210	.13119	.13119	.051	.823	0	.126	10
	1.250	38.430	.19603	.1568	.112	.636	.027	.226	11
	1.500	73.220	.27059	.1804	.179	.456	.031	.334	12
	1.750	121.873	.34910	.1995	.236	.276	.042	.446	12
	2.000	183.910	.42885	.2144	.307	.094	.049	.550	10
	2.268	369.000	.60743	.2678	0	0	0	1.000	1

[a] In efficiency sets #1 and #2, values listed under S_1, S_2, S_3, S_4 are percentages of 500 acres of tomatoes contracted or intended for sale. In set #3 these values are percentages of 1,000 acres contracted or intended for sale.

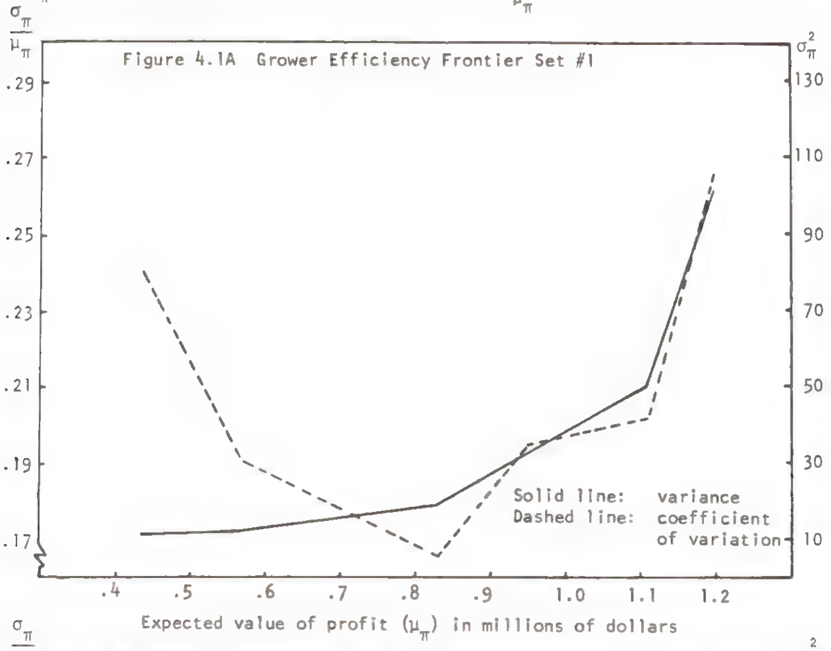
[b] These are iterations (numbers of linear subprogramming problems) required to reach a minimum-variance portfolio at each constrained-mean point.

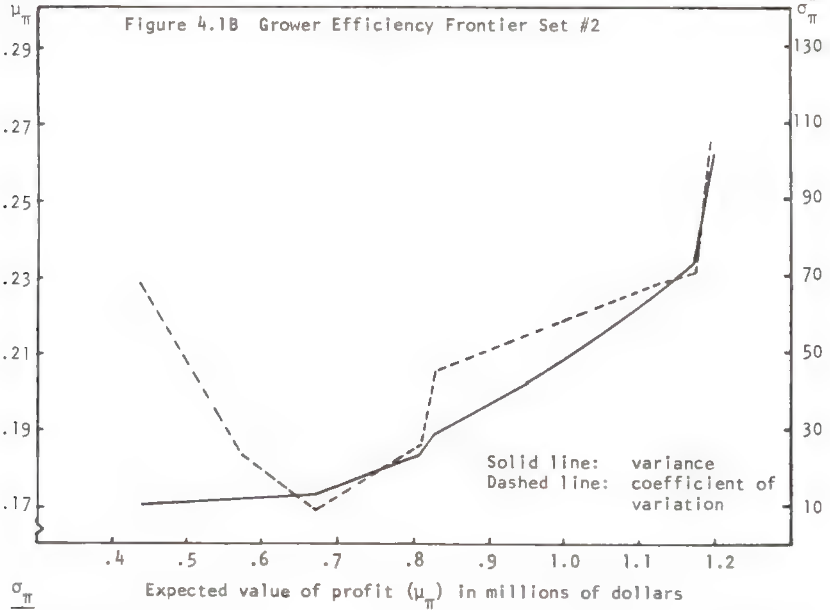
[c] The expected value, variance, and standard deviation shown here refer to the present value sum of profits over the 10-year planning horizon.

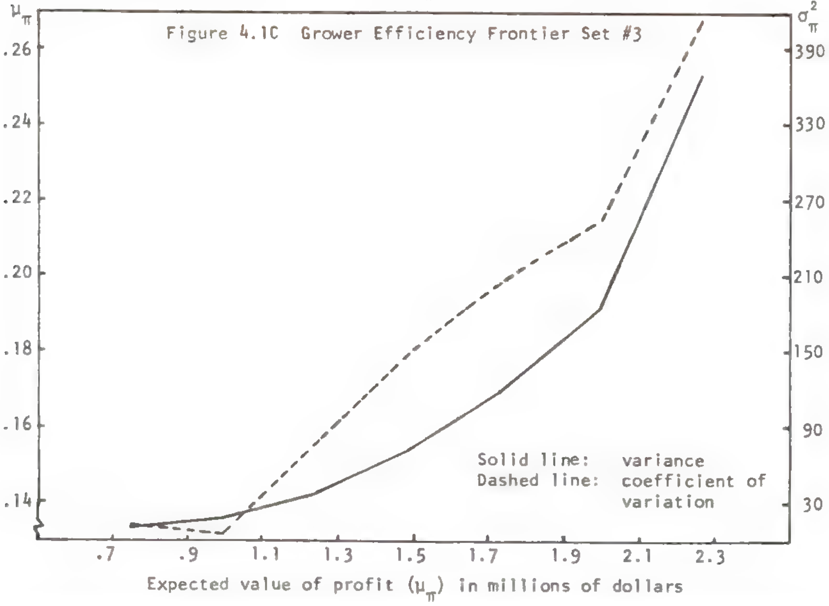
FIGURE 4.1

Grower's Efficiency Frontiers

(σ_π^2 = variance of profit in thousand trillions, $\frac{\sigma_\pi}{\mu_\pi}$ = coefficient of variation)







especially in the range over which the cost-plus option figures prominently. This in fact occurs in the lower expected profit range of E-V #2. But to the right of point .830, or $830,000 in ten-year expected profits, this frontier exhibits higher variance for the same mean values than does E-V #1. The higher variance occurs because the owner-operator cash cost of production series, in addition to having lower variance, produces weaker positive correlations with tomato market prices and tomato paste market prices (table 3.6) than does the corresponding share-lessee series.[1/]

Efficiency set #3, Figure 4.1C, further explores owner-operator portfolio options by increasing farmer tomato acreage from 500 to 1,000 acres. This greatly broadens the expected profit range from $437,000 - $1,980,000 to $746,000 - $2,268,000. The higher variance domain in #3 makes it difficult to compare the risk efficiency of alternative portfolios between #2 and #3 without reference to the coefficient-of-variation curves. Comparison of these curves indicates that the 1,000-acre farmer enjoys somewhat lower

---

[1/] Recall that the grower cash cost of production variable (VCF^a_{tom}) enters the grower profit function, Table 2.1, at two places: (a) at the cost-plus sales option, line 3, and (b) at grower cash costs themselves, line 6. A positive correlation of VCF^a_{tom} with tomato and paste market prices (the first and third sales options) translates into a positive correlation between these sales options and the second option, due to the positive sign associated with cost-plus in the profit function. But these same positive correlations translate into a negative correlation between the first and third sales options and grower cash costs due to their negative association in the profit equation.

When the cost-plus option predominates in the lower mean profit range, negative covariances in the sense of (b) above are largely inoperative; over this range the owner-operator E-V exhibits lower variances due to the lower variance of owner-operator over share-lessee cash costs. But as the cost-plus option diminishes in importance and tomato and paste market price options predominate, negative covariances in the sense of (b) above become more important. And since the share-lessee E-V exhibits higher negative covariances in the sense of (b) than does the owner-operator E-V, the former shows lower variances in the high mean profit range.

risks relative to expected profit than does the 500-acre farmer. At $800,000 expected profit over a ten-year period, for example, the smaller farmer has a coefficient-of-variation, or relative risk, of .183 and the larger farmer .134. The larger farmer also enjoys lower relative risks than the smaller farmer even when one compares, for example, the upper ranges on both farmers' curves.

If both farmers have the same attitudes toward risk taking in the sense of being willing to accept the same relative risks, one might expect the lower relative risks on the larger farmer's E-V curve to enable him to operate at a point further to the right on this curve than the smaller farmer would on his own curve. For example, if both decision makers are willing to accept a coefficient-of-variation of .2, the smaller farmer will operate at the midpoint of his E-V curve and the larger farmer will operate two-thirds of the way toward the right end of his own curve. Thus, lower relative variances accruing to the larger grower enable him to take advantage of opportunities for relatively greater expected profit or long-run earnings.

Looking at the overall results of the three E-V models, it appears that if the cost-plus markup k is 1.25 or less, it would be a very risk averse grower indeed who would rely solely on this contract option. As k increases above 1.25, the cost-plus alternative becomes more attractive. As risk aversion decreases, a mix of market price, cost-plus and coop membership may be favored. The sales-minus option assumes little importance in most solutions and so might be dropped from consideration as a practical matter. Growers who are strict profit maximizers (risk-neutral) should, under the net margin share z employed, favor 100 percent coop membership.

Cooperative Efficiency Frontiers

In this part, we develop a set of efficiency frontiers for a cooperative which purchases some of its raw tomatoes on a nonmember basis and which sells tomato paste to a distributor for reprocessing. Computational procedures are described at the beginning of this section.

Appendix A contains a listing of the mean and variance of cooperative net margin used in E-V calculations. Six variations in contract specifications are considered. The parameter values and other restrictions applicable to each set are summarized in Table 4.2. Efficiency frontier sets #1 through #5 assume that the coop has committed 25 percent of its raw product purchases to a nonmember basis. Its purchase decision is to determine the combination of market price, cost-plus, and sales-minus options that should occupy this 25 percent allocation. Nonmember purchases are reduced to ten percent in frontier set #6. Tomato paste revenues, listed under column (7) as a basis for sales-minus purchase payments to growers, assume that paste sales to distributors are evenly divided between market price, cost-plus, and sales-minus contracts.1/ A contract is modeled as signed on an acreage basis, column (8), if the tomato yield component in paste and sauce market values is random; it is modeled as signed on a tonnage basis if the yield component is constant.2/

Efficient portfolio solutions for the six contracting situations are given in Table 4.3 and the associated efficiency frontiers are graphed in

---

1/ If the sales-minus purchase option is specified so that growers are paid a share of revenue from the optimal sales portfolio, the cooperative net margin function is quadratic. The corresponding expected utility and E-V objective functions are quartic when money utilities are quadratic. Since these functions cannot be optimized by normal programming routines, a fixed revenue base for the sales-minus purchase formula is needed.

2/ This is explained more fully in Section III.

TABLE 4.2 Parameter Changes Associated with Cooperative Efficiency Frontier Sets[a]

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Efficiency frontier number	Sales side cost-plus markup (m)	Sales side sales-minus markdown (n)	Purchase side cost-plus markup (k)	Purchase side sales-minus markdown (ℓ)	Percent nonmember tomato purchases	Probability of paste market price (MP^{tr}_{pst})	Revenue base for sales-minus purchase option	Contract on acreage or tonnage basis	Contract options constrained at zero level
1	1.63	.25	1.25	.000008	25	As given in Table 3.5	All coop revenue	Acreage	None
2	1.70	.22	1.25	.000008	25	As given in Table 3.5	All coop revenue	Acreage	None
3	1.70	.22	1.25	.000008	25	As given in Table 3.5	All coop revenue	Acreage	R_3, V_3
4	1.25	.22	1.25	.500000	25	Fixed at $81.33 per ton raw equivalent	Coop paste revenue only	Tonnage	None
5[b]	1.30	.22	1.30	.400000	25	Mean, std. dev. reduced 15% from Table 3.5 values	Coop paste revenue only	Tonnage	None
6[b]	1.30	.22	1.30	.400000	10	Mean, std. dev. reduced 15% from Table 3.5 values	Coop paste revenue only	Tonnage	R_2, R_3 V_3

[a] In addition to these parameter specifications, all efficiency frontiers assume that 12,680 acres of tomatoes are contracted for purchase, and an expected 53,559 tons of paste are contracted for sale (tomato yield expectation 26.347 tons per acre).

[b] Frontiers #5 and #6 include the restriction $V_2 + V_3 \le .40$, that is nonmarket-price sales options must occupy less than 40 percent of cooperative sales portfolio.

Figure 4.2. Columns 1, 2, and 3 of Table 4.3 measure probability moments of cooperative net margin, that is the excess of cooperative revenues over cooperative nonmember raw product purchases and other expenses, prior to redistribution of these returns to the membership.

Cooperative E-V curve #1, shown on Figure 4.2A, does not exhibit the quadratic textbook pattern. After an initial negative range, risk increases with expected profit, but its rate of increase drops after the $345 million point. The associated coefficient-of-variation curve behaves similarly and even produces a second very slightly negative slope in the $346-350 million range. These results would tend to encourage the cooperative, even if moderately risk averting, to ignore the risk factor completely and act as though it were maximizing expected net margin. Points below $343 million in ten-year expected income are inefficient. In general, exceedingly risk averse coops will ignore the market price sales option and evenly divide sales between cost-plus and sales-minus. Their purchases will be weighted heavily to market price. More profit maximizing behavior encourages use of the market price sales and cost-plus purchase options.

Frontier set #2, Figure 4.2B, measures the impact of slightly altered values m, n on E-V shape and portfolio composition. The concave portion of both frontiers diminishes and is replaced by bowl-shaped functions. The market price sales option (V_1) loses its predominance in the high mean range, where it is replaced by the cost-plus option (V_2). The sales-minus option behaves similarly as before. Consequent changes in efficient combinations of the purchase options are interesting; the market price purchase option increases its importance in the middle ranges of the mean at the expense of the cost-plus purchase option. Both of these options change their portfolio percentages much more abruptly, from one mean value to another, than in frontier set #1. Since there were no changes between set

FIGURE 4.2

Cooperative's Efficiency Frontiers

(σ^2_π = variance of profit in thousand trillions, $\frac{\sigma_\pi}{\mu_\pi}$ = coefficient of variation)

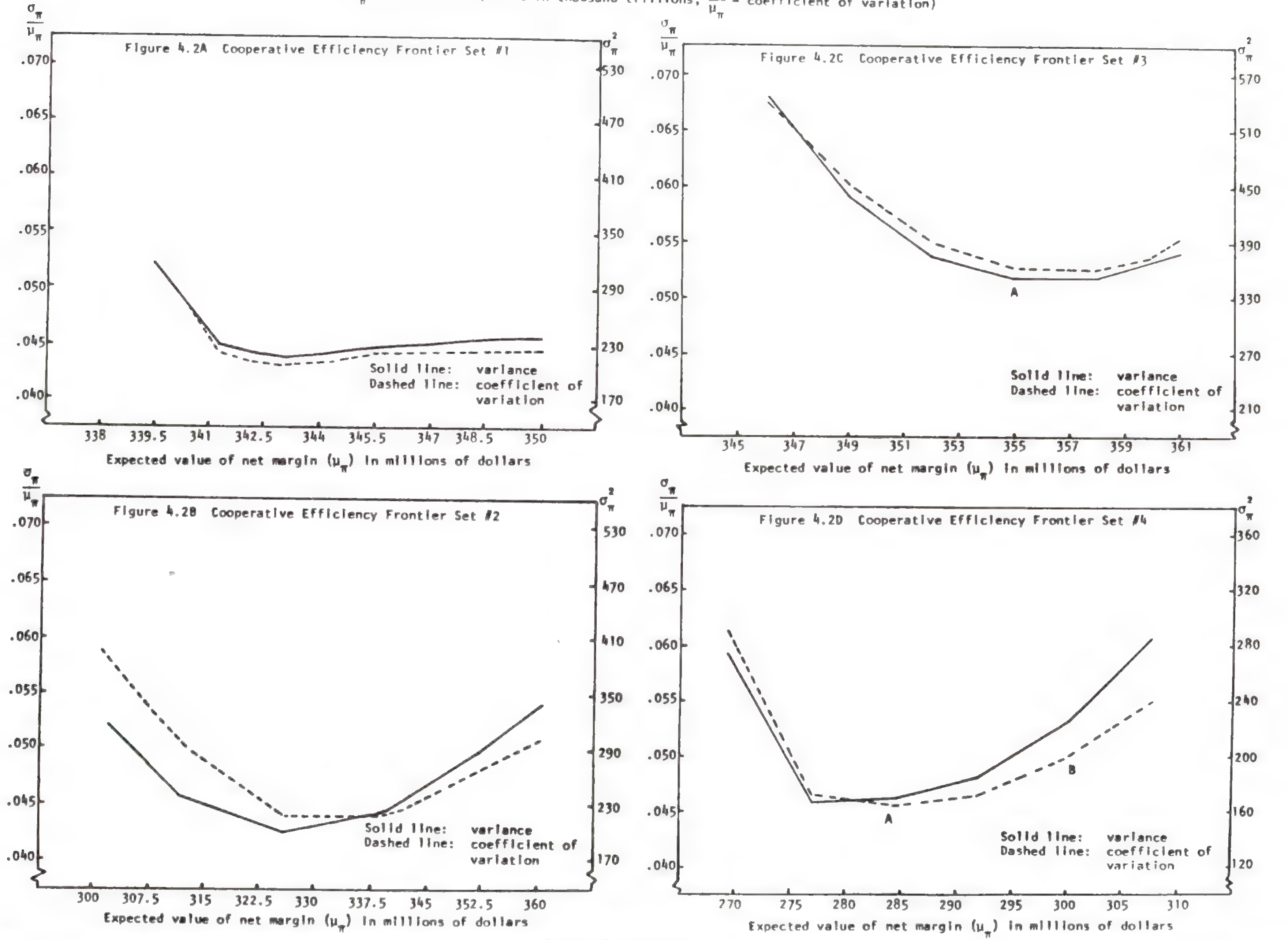


Figure 4.2A Cooperative Efficiency Frontier Set #1

Figure 4.2B Cooperative Efficiency Frontier Set #2

Figure 4.2C Cooperative Efficiency Frontier Set #3

Figure 4.2D Cooperative Efficiency Frontier Set #4

(Figure 4.2 continued)

Figure 4.2 continued

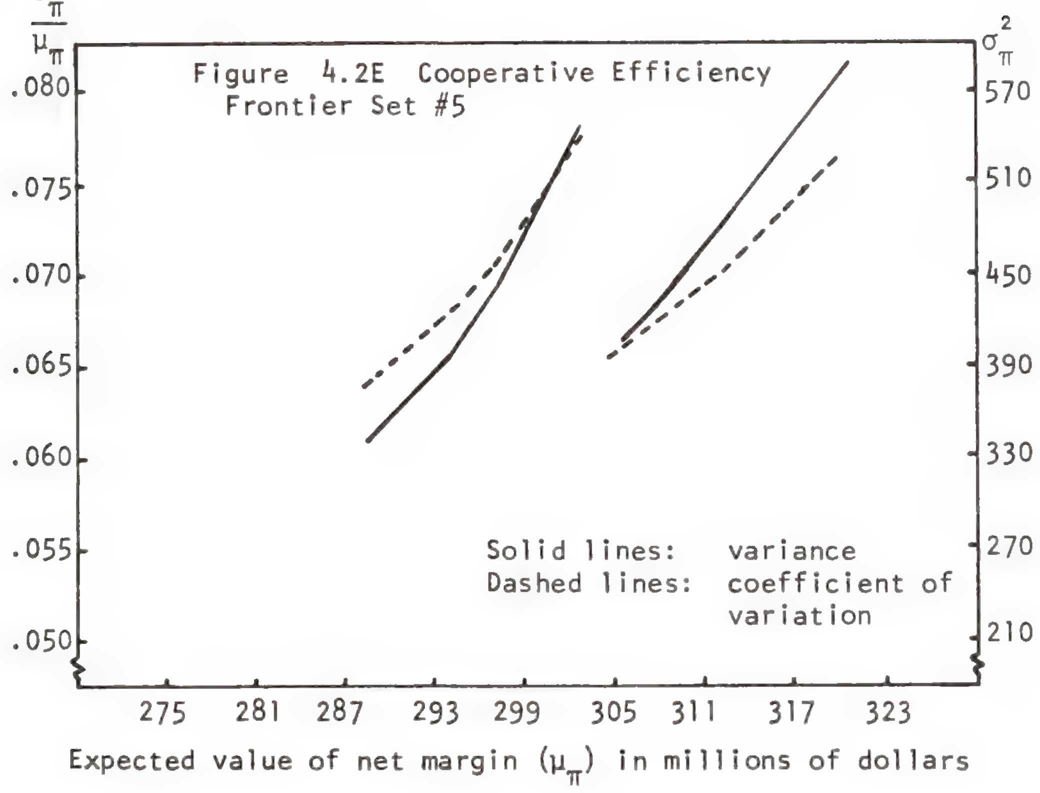


Figure 4.2E Cooperative Efficiency Frontier Set #5

TABLE 4.3 E-V and Coefficient of Variation Frontiers Indicating Efficient Portfolios of Market Price, Cost-Plus, and Sales-Minus Contracts for Cooperative Tomato Paste Sales and Raw Tomato Purchases

Efficiency set number	Moments and coefficients of net margin				Tomato paste sales[a/]			Tomato purchases[a/]			Number of iterations[b/]
	Expected net margin[c/]	Variance[c/]	Standard deviation[c/]	Coefficient of variation	Market price (V_1)	Cost-plus (V_2)	Sales-minus (V_3)	Market price (R_1)	Cost-plus (R_2)	Sales-minus (R_3)	
	million dollars	billion dollars	million dollars		proportions of portfolio			proportions of portfolio			
1	339.620	316,344	17.786	.0524	0	0	1.000	0	0	.250	2
	341.330	229,683	15.155	.0444	0	.448	.552	.105	0	.145	16
	343.049	218,305	14.775	.0431	0	.519	.481	.229	.021	0	16
	344.768	228,562	15.118	.0438	.009	.485	.506	.116	.134	0	28
	346.000	234,707	15.320	.0443	.064	.439	.497	.045	.205	0	31
	349.920	239,022	15.460	.0442	1.000	0	0	0	.250	0	2
2	301.768	316,468	17.789	.0589	0	0	1.000	0	0	.250	2
	312.000	243,655	15.609	.0500	0	.174	.826	.112	0	.138	13
	325.480	212,032	14.561	.0447	0	.404	.596	.250	0	0	2
	337.340	223,015	14.933	.0443	0	.627	.373	.250	0	0	2
	349.000	270,347	16.442	.0471	.052	.805	.143	.250	0	0	16
	356.000	308,866	17.574	.0494	.300	.668	.032	0	.250	0	14
	361.000	378,861	19.464	.0539	0	1.000	0	0	.250	0	2
3	346.019	546,199	23.871	.0675	1.000	0	0	.250	0	0	1
	347.000	512,030	22.628	.0652	1.000	0	0	.191	.059	0	1
	349.000	437,800	20.924	.0600	1.000	0	0	.060	.190	0	1
	352.000	373,032	19.314	.0549	.814	.186	0	0	.250	0	1
	355.000	349,812	18.703	.0526	.544	.456	0	0	.250	0	1
	358.000	351,995	18.761	.0524	.274	.726	0	0	.250	0	1
	360.000	367,565	19.171	.0532	.094	.906	0	0	.250	0	2
	361.000	378,861	19.464	.0539	0	1.000	0	0	.250	0	1
4	269.30	274,659	16.573	.0615	0	1.000	0	.020	0	.230	2
	277.00	166,730	12.912	.0466	.563	.437	0	.250	0	0	2
	284.70	168,940	12.998	.0456	.914	.032	.054	0	.098	.152	30
	292.00	185,010	13.602	.0465	.743	0	.257	0	.250	0	2
	300.10	225,129	15.004	.0499	.362	0	.638	0	.250	0	2
	307.70	286,639	16.930	.0550	0	0	1.000	0	.250	0	2

(table continued)

TABLE 4.3 continued

Efficiency set number	Moments and coefficients of net margin				Tomato paste sales[a/]			Tomato purchases[a/]			Number of iterations[b/]
	Expected net margin[c/]	Variance[c/]	Standard deviation[c/]	Coefficient of variation	Market price (V_1)	Cost-plus (V_2)	Sales-minus (V_3)	Market price (R_1)	Cost-plus (R_2)	Sales-minus (R_3)	
	million dollars	billion dollars	million dollars		proportions of portfolio			proportions of portfolio			
5	288.50	340,239	18.445	.0639	.623	.081	.296	0	0	.25	14
	289.60	345,230	18.580	.0642	.635	.073	.292	0	0	.25	14
	293.80	397,480	19.937	.0678	.764	0	.236	0	.050	.200	13
	295.00	411.834	20.294	.0687	.804	0	.196	0	.084	.166	13
	298.00	450,621	21.228	.0712	.904	0	.096	0	.167	.083	13
	302.80	549,712	23.446	.0774	1.000	0	0	.166	0	.084	2
6	305	405,100	20.1270	.0659	.677	.323	0	.100	0	0	1
	307	423,342	20.6480	.0672	.723	.277	0	.100	0	0	1
	310	461,082	21.4728	.0692	.792	.208	0	.100	0	0	1
	314	512,777	22.6445	.0721	.884	.116	0	.100	0	0	1
	318	570,615	23.8875	.0751	.976	.024	0	.100	0	0	2
	319	586,034	24.2081	.0758	1.000	0	0	.100	0	0	2

[a/] In all efficiency sets other than #1, #2, #3, the cooperative is assumed to contract for paste sales on a tonnage basis. For these sets, values listed under V_1, V_2, V_3 are percentages of 53,559.31 tons (200,048 barrels) of tomato paste contracted; values listed under R_1, R_2, R_3 are percentages of 12,680 acres of raw tomatoes contracted for purchase from members and nonmembers. (334,080 tons of tomatoes, at 5.46 percent solids, product 53,559.31 tons paste 32 percent.) In efficiency sets #1, #2, #3, the coop contracts for paste sales on an acreage basis. For these sets, values under V_1, V_2, V_3, R_1, R_2, R_3 are percentages of 12,680 acres of tomatoes.

[b/] These are the iterations (number of linear subprogramming problems) required to reach a minimum-variance portfolio at each constrained-mean point.

[c/] The expected net margin, variance, and standard deviation shown here refer to the discounted sum of net margin over the 10-year planning horizon.

#1 and #2 in the purchase side parameters k, ℓ, we infer that these purchase strategy differences are due to changed covariances between revenue and cost terms in the cooperative profit function. Specifically, an increase in m accentuates the importance of the strong positive covariance between cooperative processing costs (with portfolio proportion V_2) and the market price of raw tomatoes (with portfolio proportion R_1). Recall that nearly 65 percent of coop processing costs are attributed to the raw tomato market price itself. Since V_2 and R_1 are related by a negative sign in the cooperative profit function, this positive covariance translates into a negative covariance in its effect on profit variance. Thus, as V_2 increases in portfolio importance due to a rise in m, R_1 increases its importance as well.

E-V frontier #3 is defined only over the upper expected profit range of E-V #2 and exhibits uniformly higher variances than E-V #2 over this range. These effects illustrate the impact of removing options from the portfolio possibility set. The removal of any option included in an efficient portfolio set causes the variance of that set, over the range where the option was included, to rise. In addition, if the indicated option had figured prominently at either end of the efficiency curve, a portion of this end is truncated upon the option's removal since the range of feasible expected returns necessarily declines. The cooperative in E-V #3 must pay two prices for removing from consideration sales-minus formula V_3, R_3: low risk, low gain possibilities in the range \$301.77 - \$346.02 million are erased, and the remaining profit expectations \$346.02 - \$361.00 million are associated with significantly increased risk.

In frontier set #4, the sales-minus options are restored, but the market price sales option is changed to a fixed price option, defined as 25 percent less than the expected market price. The 25 percent reduction is a compensation for the reduced risks afforded under this formula. The expected profit range in E-V #4 is lower than in previous frontiers due to this reduction and the lower value m assumed. The fixed price option is most prominent among efficient portfolios in the middle expected profit range. High expected gain portfolios are dominated by the restored sales-minus sales option. However, these portfolios are associated with sharply increasing risks and consequently would be adopted by only slightly risk averse or risk neutral individuals.

The means and standard deviations of all market price variables in sets #5 and #6 are reduced 15 percent; this reduction mirrors a less bullish outlook on future tomato industry price movements than that assumed in sets #1 through #4, and corresponds to the price probability distributions evoked by several industry spokesmen.

Frontier #5 has been constructed to test the effects of limiting sales on nonmarket-price terms: the sum of cost-plus and sales-minus sales cannot exceed 40 percent of the sales portfolio. This constraint is meaningful if the latter terms represent contract sales to one customer, the market price option represents uncommitted or uncontracted sales, and the coop wishes to limit its sales to this customer for fear of becoming overly dependent upon it. Unlike the removal of entire pricing options V_3, R_3 in set #3, a linear inequality constraint such as $V_2 + V_3 \leq .40$ does not shift any portion of a frontier upwards. The constraint's presence merely removes a portion of the frontier, for which $V_2 + V_3 > .40$, from the feasibility set. Where the constraint is not violated, there is no disruption in the possibilities for portfolio mix and thus no change in the mean, variance trade-offs. In this case, the requirement that market price sales not fall below a preassigned limit has removed the lower risk portfolios from the firm's choice set. The remaining portion is steeply sloped. Moderately to strongly risk averse decision makers would choose the minimum feasible variance option and sell 40 percent of their goods on cost-plus and sales-minus contracts.

Only market price and cost-plus sales options for tomato paste may be considered in frontier set #6, and all tomatoes must be purchased at the market price. No variance minimization occurs in this situation; there is only one combination of V_1 and V_2 which will satisfy each fixed value of the net margin expectation given that $R_1 = .10$.[1] Thus there is no scatter of inefficient portfolios above E-V #6 in Figure 4.2E, and portfolio shares V_1, V_2 change proportionately to the net margin expectation assumed. Because cost-plus sales are restricted to less than 40 percent of portfolio, the lower half of E-V #6 is truncated and the firm's policy makers may only choose among relatively risky contract combinations.

Distributor Efficiency Frontiers

Mean and variance functions utilized in distributor E-V analyses are given in Appendix A. The distributor efficiency frontiers pertain to a reprocessor assumed to purchase or contract for 126,607 tons of tomato paste. With tomato yield set at the ten-year average of 26.347 tons per acre, this requires 30,000 acres of tomato land annually. Four variations in contract terms are considered as specified in Table 4.4. Efficient portfolio solutions for the four contracting situations are given in Table 4.5 and the associated efficiency frontiers are graphed in Figure 4.3.

Efficiency frontier #1, shown on Figure 4.5A, assumes that the cost-plus markup paid by the distributor to its cooperative supplier is 1.63, or

---

1/ In general, a programming problem with n variables and n independent equality constraints involving these variables has no degrees of freedom; thus, no optimization process occurs since only one solution is feasible. Here the variables are V_1, V_2, and the constraints are $\mu_\pi = \mu_\pi^o$; $V_1 + V_2 = 1$.

TABLE 4.4 Parameter Changes Associated with Distributor Efficiency Frontier Sets

Efficiency frontier number	Cost-plus markup (m)	Sales-minus markdown (n)	Probability moments of market values MV^a_{sce}, MV^a_{pst}, MV^a_{tom}
1	1.63	.258	a/
2	1.25	.258	a/
3	1.63	.258	b/
4	1.50	.250	c/

a/ As shown in Table 3.5.

b/ Mean of MV^a_{pst} reduced 15 percent; variance of MV^a_{pst} set equal to zero.

c/ Means and standard deviations of each variable reduced 15 percent from values listed in Table 3.5.

TABLE 4.5 E-V and Coefficient of Variation Frontiers Indicating Efficient Portfolios of Market Price, Cost-Plus, and Sales-Minus Purchase Contracts for a Reprocessor of Bulk Tomato Paste (Distributor)

Efficiency set number	Moments and coefficients of profit				Tomato paste purchases		
	Expected profit[b/]	Variance[b/]	Standard deviation[b/]	Coefficient of variation	Market price (W_1)[a/]	Cost-plus (W_2)[a/]	Sales-minus (W_3)[a/]
	million dollars	billion dollars	million dollars		---------proportions of portfolio---------		
1	675	10,356,770	101.768	.1508	0	0	1.000
	678	10,682,329	103.355	.1524	.218	0	.782
	681	11,188,011	105.773	.1553	.437	0	.563
	685	12,141,808	110.189	.1609	.728	0	.272
	687	12,738,516	112.865	.1643	.875	0	.125
	690	14,240,449	119.333	.1729	.784	.216	0
	693	17,225,690	131.246	.1894	.277	.723	0
	694.6	19,341,440	139.073	.2002	0	1.000	0
2	675.0	10,356,760	101.768	.1508	0	0	1.000
	702.5	11,290,620	106.257	.1513	0	.141	.859
	730.0	12,291,300	110.866	.1519	0	.283	.717
	757.5	13,358,630	115.599	.1526	0	.424	.576
	785.0	14,492,610	120.385	.1534	0	.565	.435
	812.5	15,693,250	125.272	.1542	0	.707	.293
	840.0	16,960,530	130.232	.1550	0	.848	.152
	869.6	18,397,520	135.637	.1560	0	1.000	0
3	675	10,356,770	101.768	.1508	0	0	1.000
	678	11,433,760	106.929	.1577	0	.152	.848
	681	12,618,490	112.332	.1650	0	.305	.695
	685	14,344,550	119.769	.1748	.168	.391	.440
	687	15,252,180	123.500	.1798	.288	.410	.302
	690	16,667,400	129.102	.1871	.466	.438	.100
	693	18,234,820	135.036	.1949	.277	.723	0
	694.6	19,341,440	139.073	.2002	0	1.000	0
4	354.5	9,772,219	98.854	.2789	1.000	0	0
	357.5	9,226,746	96.056	.2687	.652	0	.348
	360.5	8,934,763	94.524	.2622	.305	0	.695
	363.5	9,068,050	95.226	.2620	0	.033	.967
	366.5	10,672,984	103.310	.2819	0	.304	.696
	369.5	12,507,410	111.836	.3027	0	.575	.425
	372.5	14,571,327	120.712	.3241	0	.846	.154
	374.2	15,845,900	125.880	.3364	0	1.000	0

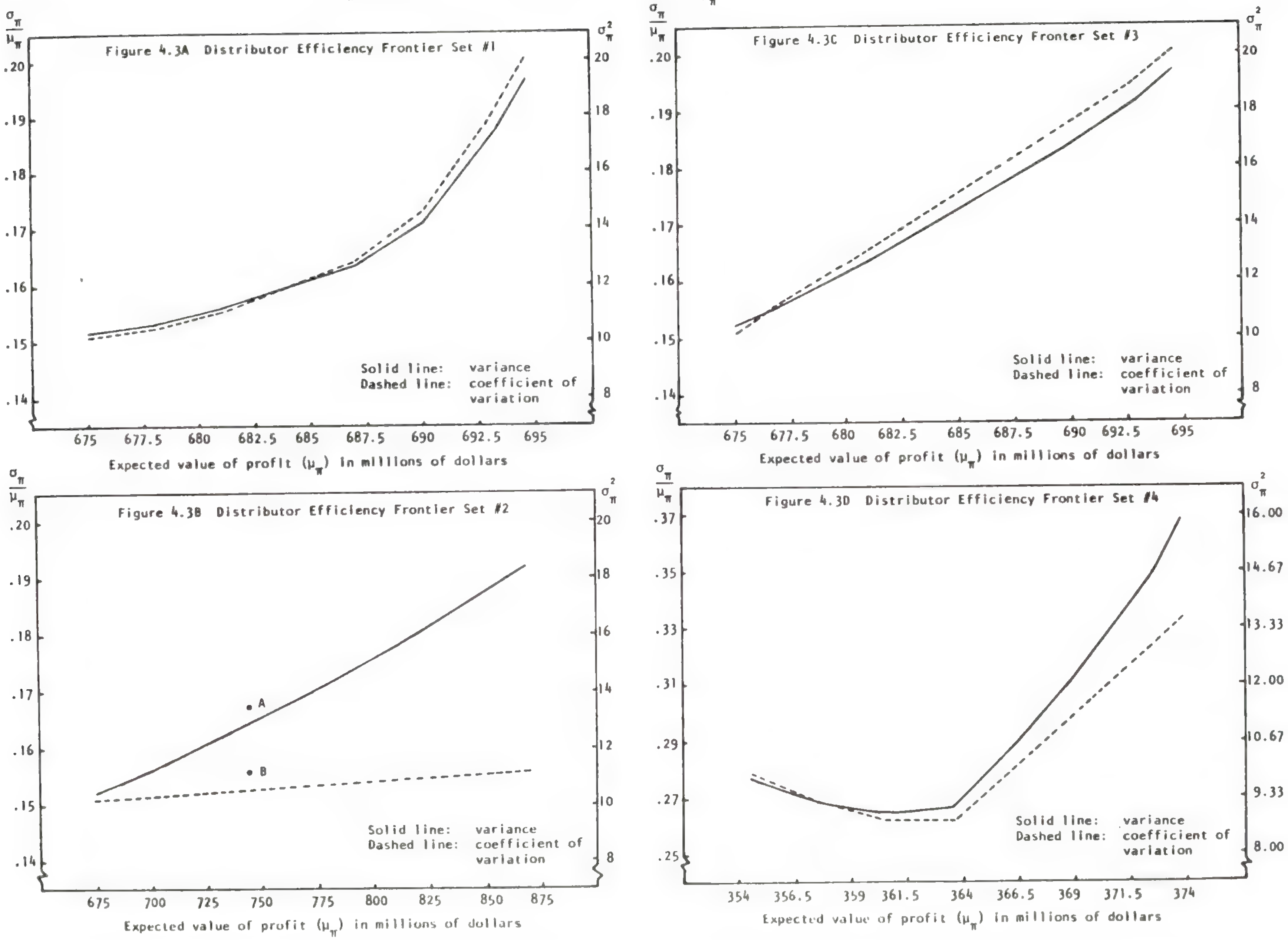
[a/] Values listed under W_1, W_2, W_3 are percentages of 126,607 tons of bulk-packed tomato paste contracted for purchase. This represents 790,410 tons of raw tomatoes, or 30,000 acres at the 1975-1984 yield expectation of 26.347 tons per acre.

[b/] The expected profit, variance, and standard deviation shown here refer to the expected value of the present value sum of profits over the 10-year planning horizon.

FIGURE 4.3

Distributor's Efficiency Frontiers

(σ_π^2 = variance of profit in thousand trillions, $\frac{\sigma_\pi}{\mu_\pi}$ = coefficient of variation)



163 percent of cooperative cash costs of bulk paste production. The sales-minus markdown is similarly 25.8 percent, the proportion of the distributor's revenue from tomato sauce sales paid to the cooperative for purchase of paste. As seen in Table 4.5, the sales-minus option predominates in the low risk, low gain portion of the E-V curve, market price in the middle range, and cost-plus in the high risk, high gain region. There are no efficient combinations of cost-plus and sales-minus options, but the market price is advantageously used in conjunction with each of these.

The effect of changing the cost-plus markup in frontier set #2 is dramatic. The E-V curve leaves its classical, approximately quadratic shape and becomes nearly linear. Increased attractiveness of the cost-plus option renders the market price option inefficient at every point, so that portfolios include cost-plus and sales-minus only. The range of possible profit expectations increases substantially.

In frontier set #3, Figure 4.3C, the market price option is changed to a fixed price offer for the duration of the contract. This price is set at a level 15 percent lower than the average expected market price over the ten-year planning horizon to compensate for the reduced risk this provides the distributor. Since the price offer is fixed, its variance and all associated covariances are zero. E-V #3 is much more linear in shape than E-V #1, Figure 4.3A, but the average slopes of the two curves are identical; that is, plotted on the same scale, their beginning and end points are in the same place. If all probabilities are normally distributed, frontier #1 is preferred to #3 at all other points than these end points since they represent lower profit variance for the same profit expectation.

It will be noted in Table 4.5 that frontier #3 is the only distributor frontier for which some portfolios include all three contract options. This is undoubtedly due to the elimination in this scenario of the paste market price variance. The effect of removing the variance is to erase all covariances that involve the market price option, namely the moderate positive covariance between the paste and sauce market prices, and the moderate positive covariance between the paste market price and paste production costs.[1/] Risk minimizing programs tend to avoid combinations of contract options with positive covariances. Thus, as those covariances decline, the associated options are more likely to be found in three-way combinations over wide ranges of the efficiency frontier.

The 15 percent reduction in the price moments in frontier set #4 is intended to represent a more bearish market outlook for these prices over the next ten years. The most immediate result of these changes, Table 4.5 and Figure 4.3D, is that the frontier's feasible expected profit region declines drastically from the \$675 - \$695 million range to the \$354 - \$375 million range. Second, a portion of this range is associated with a negative E-V slope. It is irrational for a risk averter to operate in the negative range because he could adopt portfolios with higher profit expectation and lower variance. Specifically, all market price options are inefficient since they have nonzero proportion values only on negatively sloped portions of the frontier. Watchers of the coefficient-of-variation frontier will also note that this curve turns positive at a higher expected value than does the E-V curve, indicating that risk as a proportion of the expected profit does not rise until well after absolute risk begins to rise.

[1/] Since paste production cost consists of the raw tomato cost in addition to nontomato cost, the latter covariance is a combination of $\mathrm{cov}(MV^{a}_{pst}, MV^{a}_{tom})$ and $\mathrm{cov}(MV^{a}_{pst}, NTVCC^{a}_{pst})$.

Relative Risk and the Task of Interpreting Efficiency Frontiers

Reference has been made frequently in the foregoing sections to the coefficient-of-variation curves which correspond to each E-V frontier. These curves measure the ratio of the standard deviation to the mean for each point on the E-V frontiers, and thus record relative rather than absolute changes in risk from one part of the E-V curve to another.[1/]

The advantage of coefficient-of-variation curves is that, unlike E-V curves, they provide decision makers with information that is intuitively more comprehensible. It would have little meaning to a cooperative board to inform them, on the basis of Figure 4.2D, that to increase their ten-year net margin expectation from \$277 to \$284.7 million, they must accept a \$2 trillion net margin variance increase. It seems more meaningful to tell them that the augmented net margin expectation is actually associated with a decline in relative risk from .0466 to .0456. Coefficients-of-variation are smaller, more manageable numbers and have the advantage of providing decision makers with a basis for comparing changes in the two probability moments.

After a firm has become familiar with the range of coefficients-of-variation that occurs in its contract efficiency sets, it can develop rules of thumb for use in planning an optimal contract portfolio. For example, a possible cooperative decision rule is to prefer contract portfolios with higher expected net margin up to the point where changes in coefficient-of-variation turn positive. A cooperative employing this decision rule and facing efficiency set #4, Figure 4.2D, would select point A with associated

---

[1/] An alternative measure of relative risk is the relative variance curve (σ^2_π/μ^2_π).

sales portfolio: market price, 91.4 percent; cost-plus, 3.2 percent; sales-minus, 5.4 percent. An alternative decision rule is to select portfolios with higher expected profit up to the point where positive rates of change in coefficient-of-variation exceed some preassigned limit. Such a point might be point B in Figure 4.2D, where the preassigned limit is a 45-degree slope.[1]

The purpose of efficiency frontier analysis is to illustrate tradeoffs, for alternative efficient contract portfolios, between the mean and risk or relative risk of income. The specification of "efficient" portfolios is important. Coefficient-of-variation frontiers limit the firm's decision making problem greatly by eliminating all portfolios for which relative risk could be reduced without sacrificing profit expectation. As an example, an inefficient contract portfolio that sets each purchase option at 33.3 percent has been plotted for distributor E-V #2 on Figure 4.3B. This portfolio, if adopted under the parameter conditions specified for efficiency set #2, Table 4.4, would provide the distributor with a ten-year present value profit expectation of $744.43 million, a variance of $\$13.4 \times 10^{15}$, and a relative risk of .1555. Point A in Figure 4.3B indicates the present value mean, variance point and point B the present value mean, coefficient-of-variation point. Both of these are above their corresponding efficiency lines. A distributor employing this contract strategy would suffer a relative risk nearly identical to the all-cost-plus strategy that provides 16.8 percent higher expected profit.

---

[1] A reviewer has added the insight that efficient mean, standard deviation pairs can be combined with the Chebyshev inequality to provide, for each point on an E-V frontier, the probabilities of experiencing profits within prescribed ranges of the mean. Observing the inequality $\Pr[|\pi - \mu_\pi| \leq \Theta\sigma_\pi] \geq 1 - \Theta^{-2}$, $\Theta > 0$, the reviewer calculates, as an example, the least probabilities of obtaining a profit within $\Theta = \mu/\sigma$ standard deviations of the mean for each listed point on Grower E-V curve #1, Table 4.1. At point ($\mu = .437$, $\sigma = .10488$), Θ equals 4.167; hence at this point the least probability of obtaining a ten-year profit in the range zero to $874 million is .942. The Chebyshev inequality may also be used to indicate the range within which profits will remain for at least a prescribed probability.

A disadvantage of efficiency curve analysis, perhaps especially in portfolio problems, is its dependence upon and extreme sensitivity to parameter changes such as the cost-plus markup assumed. This sensitivity is seen by comparing distributor efficiency curves #1 and #2, Table 4.5 and Figures 4.3A,B. Here a decrease in the cost-plus markup m from 1.63 to 1.25 increases the range of ten-year expected value sums from $19.4 to $194.6 million. Consequently a new efficiency curve must be calculated whenever a new markup, markdown, or other parameter is considered.

Another disadvantage of efficiency frontiers is that there appears to be no way of employing them for the purposes of a determinate portfolio decision without some reference either to a utility function or to a rule of thumb such as those discussed above. These rules of thumb have no theoretical basis; a decision maker may be unable to discover a rule of thumb that he is certain will serve his best interests. More specifically, it is impossible to know whether a given rule of thumb will identify a contract portfolio that maximizes the expected utility of any Von Neumann-Morgenstern utilist. To accomplish this identification, it is necessary to combine information contained in E-V (not coefficient-of-variation) curves with the decision maker's money utility. We turn to a consideration of utility functions in the next section.

V. ESTIMATION OF MONEY UTILITIES

Our previous development (Section II, eq. 5 and footnote) has shown that the utility of any contractual arrangement to a particular participant may be expressed as:

$$E[U(M)] = U(\mu_m) + (1/2)\sigma_m^2\, U''(M)$$

+ higher moments for nonnormal distributions.

In the above, $U(\mu_m)$ or $U[E(M)]$ is the utility of the expected monetary result and σ_M^2 is the variance of monetary results.[1/] For grower or distributor, M is net profits. For the cooperative board of directors and management, M is cooperative net margins. If $U[E(M)]$ increased linearly with M, the higher derivatives would be zero and the utility of any contractual arrangement could be measured in terms of only its expected monetary outcome. But for risk averters or risk takers, the utility of expected monetary returns varies nonlinearly with the rate of return. In this section we describe the procedures used to estimate utility functions and present the results of these estimates. These results are then used to solve for maximum-expected-utility portfolios in Section VI. Subjects for utility estimates include growers who sell to the cooperative on a membership and nonmembership basis, and an executive and director of the cooperative itself.[2/]

1/ Utility may be influenced by other outcomes as well, such as perquisites associated with large firm size. Firm goals may also be ordered to form a lexicographic utility function [Ferguson, 1965]. Our interviews with coop personnel did not uncover these aspects of utility variation. Note that the argument M (money) in the above function is equivalent to the argument π (profit) in Section II.

2/ We were not able to obtain interviews and develop utility functions for the distributor. A synthetic distributor utility function is later developed for illustrative purposes in evaluating alternative outcomes of the long-term contracting process.

Utility Measurement

A widely known method for estimating utility function for profit or net margins is to ask a respondent to consider a gamble with given probabilities of two specified dollar outcomes [Von Neumann and Morgenstern]. It is not required that either outcome involve a loss, although losses are specified by most researchers. For each gamble, the respondent is then asked to name a no-risk dollar receipt (or payment) such that the respondent would be indifferent between taking the gamble and receiving (or paying) the no-risk amount. The respondent's utility for the no-risk amount is calculated by assigning arbitrary utility values to the two possible gamble outcomes, and invoking the expressed indifference between gamble and no-risk payment to calculate

$$(17) \quad U(M) = [U(M_1)\cdot p(M_1)] + [U(M_2)\cdot (1-p)(M_2)]$$

where M is the no-risk receipt or payment,

M_1, M_2 are the gamble pay-offs,

U ( ) is utility, and

p ( ) is probability.

Selective alteration of $p(M_1)$, $1-p(M_2)$ will cause changes in the specification of gamble, and thus in the no-risk amount to which the respondent is indifferent. Hence, a utility value can be calculated for as wide a money range as the interviewer and respondent choose.

Writers in utility theory have mentioned some important pitfalls in this procedure [Von Neumann and Morgenstern, pp. 18-19, 30-31; Luce and Raiffa, pp. 34-37]. Some decision makers may not possess a utility function which obeys the axioms invoked by Von Neumann and Morgenstern, and truthful answers of these persons to the utility questionnaire would be meaningless or difficult to incorporate into our analysis. Even if respondents obey all utility axioms, their money utilities may change so quickly over time that any conclusions one may draw from a single utility estimate are useless. It has also been warned by several writers [Luce and Raiffa, p. 36; Halter and Dean, p. 63] that subjects would be encountered who do not understand the fundamental concepts of probability, for example that the sum of probabilities is always one.[1/]

Although these clearly are important limitations, the value of even rough estimates of utility functions offsets the problems associated with their estimation and interpretation. Such functions enable us to estimate the general nature and range of risk aversion and provide a means of approximating the likely ranges of optimal contract portfolios. From these estimates we are able to develop some generalizations which seem likely to be of value to those concerned with long-term contracts.

Interview Method

The first step in the interview process was to decide upon an appropriate range of dollar values over which to define a function. Since it was considered out of bounds to inquire into current or usual profits or losses of grower respondents, a rough upper profit rate per acre was calculated and this rate was multiplied by the total number of acres which would typically be farmed in each grower's size category. The resulting dollar profit was used as the high side of the utility range. The low side was usually

---

[1/] The Von Neumann-Morgenstern procedure of utility function estimation was chosen over equi-probability (modified VN-M) and Ramsey methods because the process of altering probabilities of fixed dollar pay-offs seemed most natural for respondents to react to. Officer and Halter found that the VN-M method performed more poorly than the other two methods in predicting actual behavior of respondents. However their sample size was too small to make firm conclusions about this. The objection to VN-M that subjects are biased against "gambling" is not compelling and was borne out only in several cases in our interview process. Even where this bias occurs, interviews may be worded to remove the gambling issue.

calculated by dividing the high side by two and multiplying by -1. "Large" farmers (greater than 1,400 total acres) were assigned the annual profit range -\$300,000 to \$700,000; "medium" farmers (700 to 1,400 total acres), the range -\$150,000 to \$300,000; and "small" farmers (less than 700 total acres), the range -\$75,000 to \$150,000.[1/] Each cooperative spokesman provided utility responses for the range -\$15,000,000 to \$25,000,000 in net margins.

Once the range for each respondent was determined, the extreme high and low sides of this range were employed as the dollar pay-offs (or losses) of the proposed Von Neumann-Morgenstern gamble. The high side was assigned the arbitrary utility value 100, and the low side 0. Each respondent was then asked whether he would accept a gamble in which there was an 80 percent chance of the gain and a 20 percent chance of the loss. Pay-offs or losses were interpreted as profits or losses from each respondent's farming or processing enterprise in 1975. If the respondent declared he would be willing to accept the gamble proposed, he was offered a no-risk dollar payment instead of the gamble. No-risk payment offers were interpreted as offers to rent the farmer's or processor's land, building, and equipment in 1975 for a cash fee. The first rent offer was arbitrarily chosen, but this amount was altered until the point was found where respondents were fairly indifferent between the proposed farming gamble and the rent offer.

Care was taken that the respondent did not interpret the cash rent offer as a proposal to bargain over the terms of rent. If the respondent thought he could bargain, he would refuse any cash rent offer that he

---

1/ There were several exceptions to this rule, notably in the case of grower #6. An exception was allowed when it was discovered that a grower's annual expected profits differed significantly from the midpoint of the original range.

thought the interviewer might be willing to improve upon, regardless of whether he preferred this cash offer to the proposed gamble. Respondents were informed that bargaining over cash rents was out of place and were asked to consider each cash offer as the "last" offer.

Once the cash rent indifference point was found for the 80-20 gamble, the respondent was asked to consider a gamble identical to the above in all respects, except that odds are changed to 60 percent chance of winning and 40 percent chance of losing. If the respondent was willing to accept this gamble outright, he was offered a cash rent in its stead and a gamble, cash rent indifference point was found. If he was not willing to accept the gamble outright, he was informed that a penalty would have to be paid to avoid the gamble. The size of penalty for which the subject was indifferent between penalty and gamble was then determined similarly to the gamble, cash rent indifference point, care being taken to ensure that the penalty level was not a subject for bargaining.

Similar procedures were repeated for gambles with 40-60 odds and for 20-80 odds. In all, the cash rent or penalty indifference points for four such gambles were recorded. Since the arbitrary utility assignment to the winning amount of each gamble was 100, and to the losing amount zero, it was convenient to apply equation (17) to each gamble in order to calculate the farmer's or processor's utility for the corresponding no-risk payment or penalty. For example, if the cash rent indifference point to an 80 percent chance of $300,000 and 20 percent chance of -$150,000 is $10,000, the subject's utility for $10,000 is (.8)(100) + (.2)(0) = 80. The subject's utility for the no-risk indifference point in the 60-40 gamble is invariably 60, and so forth.

Most respondents were surprised at the questions posed to estimate their money utility and found it hard work to provide answers. It is clear that most of them, especially the older ones, were not used to thinking in terms of abstract probabilities and abstract or hypothetical business situations. Reactions to the suggestion that they reason in these terms ranged from outright refusal, to discomfort or impatience, to eagerness. It is natural that we should rate the estimates from the eager respondents more highly than the others, since it is probable that the less patient ones did not give as careful consideration to their answers. Eager respondents, however, exhibited the spectrum of linear, moderately risk averse, and strongly risk averse utility.

The most important difficulty encountered was for interviewer and respondent to locate the proper level of abstraction from which meaningful answers could be drawn. Most growers and cooperative representatives objected that the reference gambles were too hypothetical. It seemed to them useless to consider an 80 percent chance of losing money when the normal risk of such a loss is only five percent or ten percent. Others objected that real-life risk situations do not carry known probabilities, as our reference gambles seemed to imply.

Another abstraction problem arose over the business situation represented by the reference gambles. Two growers' biases against gambling complicated the interview process; these respondents would not answer questions in which "gambles" were mentioned because of bad connotations with dice and cards. When, to discourage this bias, the reference contract was expressed in terms of a one-shot farm-or-lease-out decision, it became unclear how strictly this analogy should be taken. Several growers objected that leased equipment would always be returned worn or broken (the no-risk cash alternative). Others were biased against leasing because of personal commitments to laborers, suppliers, and landlords. They were assured that the lessee would honor these personal obligations. Another problem was the difficulty some had accepting a situation in which they had to pay cash to avoid a gamble they considered undesirable; reactions that this situation constitutes extortion tended to bias utilities toward risk seeking in the lower dollar range.

Interview Results

We were able to obtain interviews with nine grower members of the cooperative, a top management executive, and a director. Information on the attitude toward risk of the total board of directors was also developed in a manner to be discussed presently.[1/] The growers were selected to cover a range of characteristics (see Table 5.1) which we hoped could be related to their attitudes toward risk.[2/] Our objective was to obtain representative utility functions for the various participants.

Growers' responses to the money utility interview procedure are recorded on Table 5.2 and graphed in Figure 5.1. Grower #5 refused to answer the questionnaire. Grower #6 on the other hand volunteered two sets of responses corresponding to widely differing wealth ranges. These are listed as #6A and #6B.

Utility responses of the cooperative executive and director are given in Table 5.3 and graphed in Figure 5.2. The responses suggest that the executive is indifferent between a $3,000,000 payment and a gamble whose pay-offs to the cooperative are $25,000,000, -$15,000,000, regardless of the odds associated with these pay-offs. This attitude is only marginally

---

[1/] As noted earlier, we were not able to obtain interviews with distributor representatives. A synthetic function was developed for illustrative purposes, as explained later.

[2/] A larger sample would have been desirable for this purpose. However, the time and difficulty of obtaining interviews precluded our contacting more respondents.

TABLE 5.1 Summary of Grower Socioeconomic Data

(1) Grower number	(2) Business status	(3) 1974 acreage	(4) 1974 tomato acreage	(5) Proportion tomato to total acreage	(6) Proportion of 1974 acreage owned	(7) Approximate age	(8) Years farming	(9) Years in cooperative membership	(10) Number of customer canners	(11) Proportion of 1974 acreage in cooperative membership
1	brothers	6,000	700	.12	0	55	27	27	1	1.00
2	brothers	1,405	950	.68	.33	45	10	?	2-3	?
3	single	1,245	500	.40	.15	45	18	7	2-3	.40
4	father - son	1,200	460	.38	.13	55, 25	15, 2	nonmember	4	0
5	single	1,200	850	.71	.33	50	20	?	1	1.00
6	father - son	870	200	.23	.13	80, 45	?	36	1	1.00
7	single	790	500	.63	.30	40	15	?	3	.40
8	single	660	150	.25	.61	45	22	?	1	1.00
9	father - son	565	120	.21	.50	?, 28	?, 2	1	1	1.00

TABLE 5.2 Grower Responses to Utility Function Questionnaires

Utiles	Grower #1	Grower #2	Grower #3	Grower #4
100	$700,000	$700,000	$300,000	$2,000,000
80	300,000	525,000	201,000	1,500,000
60	200,000	350,000	0	750,000
40	- 50,000	- 125,000	0	35,000
20	- 150,000	- 175,000	0	100,000
0	- 300,000	- 300,000	- 150,000	- 1,000,000

Utiles	Grower #6A	Grower #6B	Grower #7	Grower #8	Grower #9
100	$1,000,000	$4,000,000	$300,000	$150,000	$300,000
80	- 25,000	250,000	100,000	100,000	30,000
60	- 62,000	- 250,000	30,000	50,000	20,000
40	- 100,000	- 350,000	20,000	- 5,000	0
20	- 300,000	- 400,000	- 50,000	- 35,000	- 40,000
0	- 500,000	- 1,000,000	- 150,000	- 75,000	- 150,000

FIGURE 5.1

Money Utility Observations for the Grower Sample

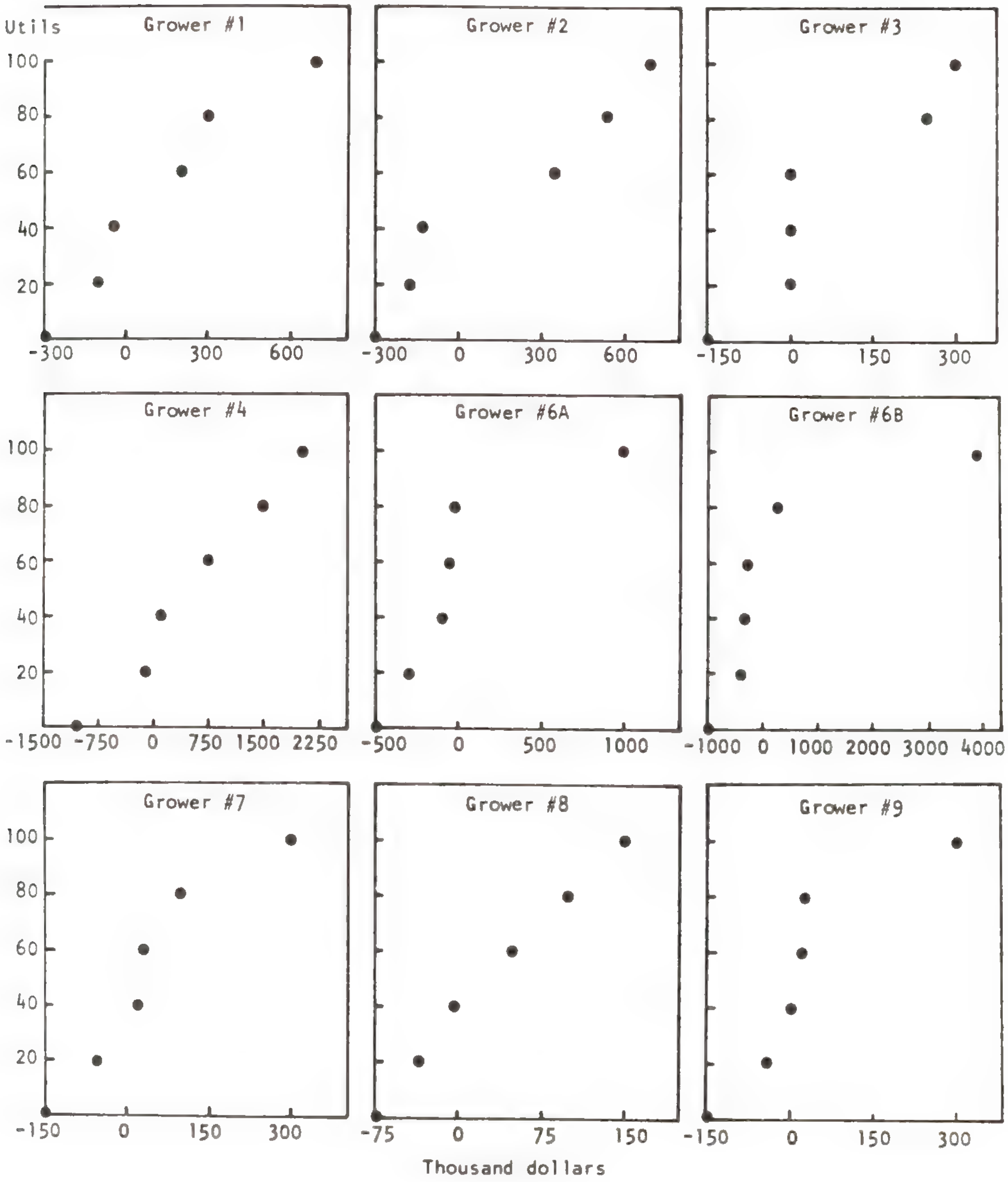


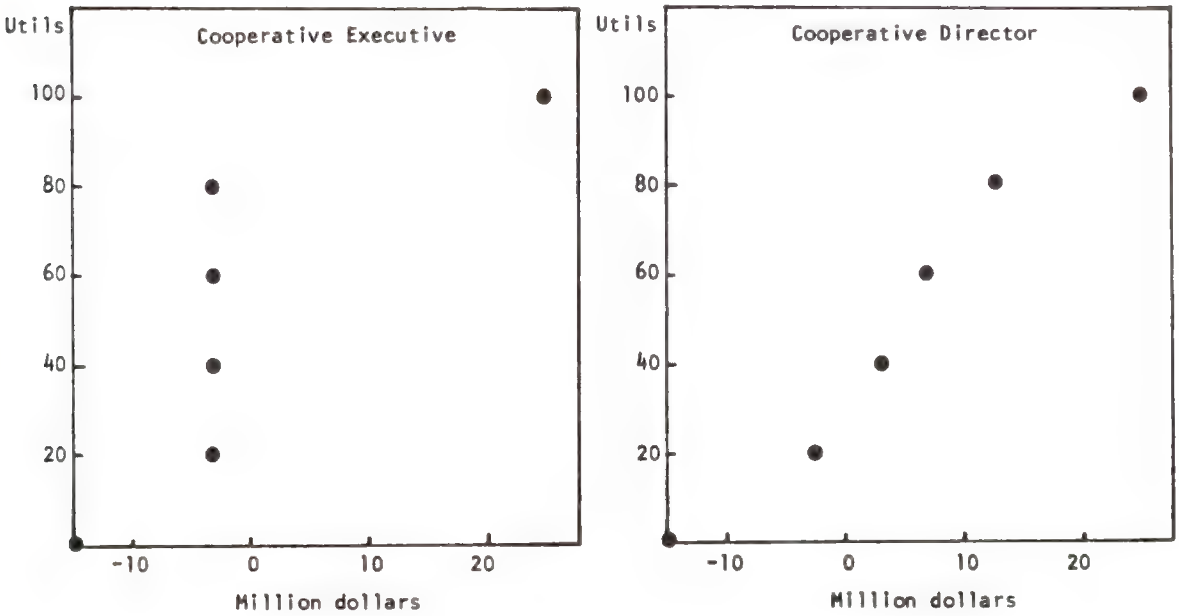
TABLE 5.3

Responses to Utility Function Questionnaires:
Cooperative Spokesmen

Cooperative executive		Cooperative director	
utils	dollars	utils	dollars
100	25,000,000	100	25,000,000
80	- 3,000,000	80	12,500,000
60	- 3,000,000	60	7,000,000
40	- 3,000,000	40	3,000,000
20	- 3,000,000	20	- 2,500,000
0	- 15,000,000	0	- 15,000,000

FIGURE 5.2

Money Utility Observations for the Cooperative Executive and Director



rational; ordinarily one expects utility to rise only as money income rises, and utility may not rise as income falls. When the unusualness of these answers was pointed out, the executive replied that the prospect of dollar losses in the range of \$15,000,000 would induce irrational behavior in any cooperative of similar size. In further conversation, however, he indicated that a processing cooperative's first goal is to provide a secure home for member produce. It is possible to construct a thoroughly rational marketing strategy on this basis, either through invoking a minimax criterion or through appealing to a strongly risk averse money utility function. One summary measure of the executive's feeling toward risk bearing is provided by fitting a continuous function, for which $U'(M)$ J 0, through his actual responses. Another measure, emphasizing stronger risk aversion inherent in the "secure home" statement, could be obtained by dropping the observations corresponding to 60, 40, and 20 utils. The former course was taken in this study.

The member of the cooperative board of directors who was interviewed for this study felt that a cooperative was too safety-oriented that sought only to provide a secure home for its members' products. He detected a systematic difference in business philosophy between "managers" and "entrepreneurs." Managers are particularly wary of year-end net losses, he said, because these threaten their job security. This fear prevents them from taking risks necessary to operate an optimally successful business. Entrepreneurs, on the other hand, take the risks necessary to maximize profits. This respondent classified himself as an entrepreneur both in his own farm operation and as a voice in cooperative affairs.

His answers (Table 5.3) as a cooperative director to the standard utility reference contract support this self-evaluation. Although the observations in Figure 5.2 suggest first risk seeking, then neutral, and finally risk averse behavior, the sense of the respondent's answers indicated he was attempting to describe a linear function. Each time a point of indifference between gamble and premium or penalty was sought, the director said he was looking for a dollar value which would "describe the gamble," based on its pay-offs and probabilities. This is a clear reference to the gamble's expected value. Thus deviations from the function's linear tendency may be ascribed to an imprecise notion of expected value or to hasty calculations. Our conclusion is that this board member would urge the cooperative to maximize expected profit regardless of the size of risks involved.

Whether the other board directors would agree with this urging may be found by observing the circumstances of an actual board decision. The board recently faced an expansion decision which presented a possible $4 million profit or $4 million loss. The subjective probability assigned by cooperative management to the positive pay-off was 90 percent, and to the loss, ten percent. The expected value of this bet is $3.2 million profit. However, the board rejected the proposal, that is it chose no-risk zero dollars in its place, because the proposal was considered too risky. The strong risk aversion displayed in this decision is evident by expressing it as

$$(17)' \quad U(\$0) \geq .9U(\$4 \text{ million}) + .1U(-\$4 \text{ million}).$$

When utilities 100 and 0 are assigned to a $4 million gain and loss, respectively, we have

$$\begin{aligned} U(\$4 \text{ million}) &= 100 \\ U(\$0) &\geq .9(100) + .1(0) = 90 \\ U(-\$4 \text{ million}) &= 0. \end{aligned}$$

If the middle inequality is replaced by an equality, the utility relationship contradicts the linearity of the director's function and corroborates the moderate-to-strong risk aversion implicit in the executive's responses.

Estimates of Utility Functions

Each of the sets of utility observations may be regarded as a sample pertaining to the respondents' utility scale for money. Since the responses are intuitive they do not necessarily all fall precisely on the line which would represent the "real" attitude toward money and risk.[1] In order to solve for optimal contract portfolios we need to estimate functions which best describe the utility-money relationships and at the same time are tractable for further mathematical analysis.

Two functional forms were explored: quadratic ($U = a + bM - cM^2$, $b, c > 0$) and negative exponential ($U = K - \Theta \exp[-\lambda M]$, $\Theta, \lambda > 0$). Both have the desirable property of yielding optimal solutions by quadratic programming routines.[2] As noted by Arrow, the quadratic function has the theoretical property, considered undesirable by some, of increasing absolute risk aversion with greater income. The exponential function is an improvement in the sense of exhibiting constant absolute risk aversion, but is not necessarily a better fit to the data over the range of observations.

Our procedure was to fit quadratic functions to all of the utility data sets and exponential functions to selected sets for comparative purposes, the latter being somewhat more difficult to estimate. Either function must be viewed as an approximation valid only over the range of observation and, in the case of the quadratic function, restricted to the increasing portion of the curve. In application we shall be concerned with functional properties over only fairly narrow ranges of expected profit levels appropriate to the problem under consideration. At expected income points where both functions show about the same degree of risk aversion they may be expected to yield somewhat similar portfolio solutions.

---

[1] If the interview were repeated at another time or perhaps formulated in different terms, we might expect a different scatter of dots, but falling within the same general pattern. Replication of this sort was not possible.

[2] We also explored cubic functions but the improvement in fit, if any, was not sufficient to justify the added mathematical complexity in obtaining optimal solutions.

Quadratic Functions

The quadratic utility functions are presented in Table 5.4. The quadratic terms (c coefficients) are not significantly different from zero for growers #2, #3, and #4, suggesting that their utility functions are linear. Thus, they appear risk-neutral over the observed range. This is evident by inspection of Figure 5.1.[1/]

The quadratic terms are also statistically nonsignificant for both the coop executive and the director. In the case of the executive, Figure 5.2 strongly suggests concavity and the lack of significance is due to the wide scatter and limited observations. The negative quadratic term thus seems acceptable in this case, reflecting a moderate degree of risk aversion. For the director, a cubic function actually fits the data better, suggesting first risk-seeking, then risk-neutral, and finally risk-averse behavior. However, it was noted earlier that the overall attitude of the respondent seemed more consistent with risk neutrality. The function for the board of directors was not obtained by a statistical fit but was derived by imposing

---

1/ An attempt was made to link differences in risk aversion among growers to their socioeconomic characteristics by regressing coefficients of relative risk aversion as dependent variable against selected variables in Table 5.1. The coefficient of relative risk aversion [Arrow, Pratt], $-MU''(M)/U'(M)$, provides a basis for comparing risk aversion of different subjects and at different income levels; they were here evaluated at the midpoints of each quadratic utility function. Socioeconomic attributes tested were 1974 acreage, proportion of 1974 acreage owned, age, and number of customer canners. There were no statistically significant results, possibly owing to the small sample size available.

TABLE 5.4 Estimates of Quadratic Money Utility Functions, $U = a + bM + CM^2$ a/

Respondent	Profit range ($000)	a	b	c	R^2	$R_r(M)$ b/
Grower #1	- 300 to + 700	43.04 (16.29)	1.265 (12.53)	- .0064 (-3.20)	.989	.2537
Grower #2	- 300 to + 700	36.96 (4.74)	.916 (3.77)	- .0011 (-.195)	.947	.0504
Grower #3	- 150 to + 300	39.51 (4.475)	2.392 (3.279)	- .0136 (-.409)	.885	.0932
Grower #4	- 1,000 to + 2,000	32.15 (8.40)	.333 (7.79)	.000012 (.040)	.980	.0036
Grower #6A	- 500 to + 1,000	66.13 (7.75)	1.069 (5.009)	-.00725 (-2.55)	.917	.5131
Grower #6B	- 1,000 to + 4,000	63.07 (7.988)	.565 (4.632)	-.00118 (-3.418)	.925	1.6777
Grower #7	- 150 to + 300	44.72 (7.95)	2.865 (5.97)	-.0317 (-1.51)	.946	.2000
Grower #8	- 75 to + 150	38.75 (25.58)	4.751 (22.335)	-.0487 (-2.26)	.997	.0833
Grower #9	- 150 to + 300	50.76 (5.86)	3.098 (3.82)	-.0469 (-1.39)	.872	.2938
Cooperative executive	-15,000 to +25,000	59.82 (3.99)	.031 (2.36)	-.000006 (-.85)	.714	.240
Cooperative director	-15,000 to +25,000	36.47 (7.30)	.027 (7.14)	0 (0)	.962	0
Cooperative board of directors	- 4,000 to + 4,000	90	.125	-.00025	--	.250

a/ Money is expressed in $10,000 units. To express in $1 units, move decimals in regression coefficients to the left: 4 places for b coefficients, 8 places for c coefficients. The a coefficient is without behavioral significance and reflects only the arbitrary utility scale selected. Values in parentheses are t-ratios.

b/ $R_r(M)$ is coefficient of relative risk aversion, $-MU''(M)/U'(M)$, here evaluated at the midpoint of each quadratic function, except for the board of directors. The latter was evaluated at $500,000. The midpoint of the board's function is a $0 where $R_r(M)$ is also zero.

a quadratic function on the points generated from the observed action of the board described earlier.

Exponential Functions

Negative exponential functions were fitted to the data for growers #1 and #6A, the cooperative executive, and the cooperative board. Finding a best-fit exponential function requires substantially more effort than the OLS fit of a quadratic equation. The procedure employed here was to move the constant K in this function to the left hand side to give

(18) $U - K = -\theta e^{-\lambda M}$.

In order for utility to be positive, K must be positive and greater than U; thus $(U - K) < 0$. Multiplying by -1, we obtain

(19) $-U + K = \theta e^{-\lambda M}$.

Taking natural logs of both sides,

(20) $\ln(-U + K) = \ln\theta - \lambda M$.

The OLS estimate of this relationship yields $\ln\theta$ as a constant term and $-\lambda$ as coefficient of money. The anti-log of $\ln\theta$ and negative of $-\lambda$ may then be substituted into (18). Since estimates of θ and λ depend on the value of K chosen, it is necessary to try a set of K's and choose that corresponding function which best fits the original data.[1] Such fits are hand calculated by minimizing the sum of squared errors.

The exponential fits to the data are as follows:

Grower #1 $U = 160 - 117.3e^{-.001002M}$

Grower #6A $U = 120 - 60e^{-.001194M}$

---

[1] Log utility values here represent nonlinear utility transformations; hence maximizing R^2 from the log fits is not a legitimate selection criterion.

Cooperative Executive $U = 120 - 59e^{-.001194M}$

Cooperative Board $U = 101 - 10.4e^{-.5769M}$.

In the above, M is measured in \$10,000 units. As is noted in Arrow or Pratt, the superscript coefficient λ is a measure of the constant absolute risk aversion.

Quadratic and Exponential Functions Compared

If the quadratic and exponential function for Grower #1 were graphed in Figure 5.1, they would lie close together over the range of observations. Either would be an acceptable fit to the data. For grower #6A, however, the two results differ significantly. Because of the peculiar pattern of observations, the quadratic function reaches a peak at \$700,000 and then declines - a logically unacceptable result. The logistic function avoids this problem but seems to underestimate substantially the degree of risk aversion in the upper ranges. We would have little confidence in either function as a reliable estimator of the utility relationship for this grower.

Turning to the cooperative executive, the two functions are similar over most of the range of the data, particularly in the positive part. For the board of directors, the quadratic and exponential functions are almost identical over the observed range of -\$4 million to +\$4 million. Although extensions much beyond the observed range would be questionable with either function, the exponential curve would appear more acceptable for larger values of M since the quadratic would eventually peak and then turn down.[1]

---

[1] In comparing the executive and board functions and the different ranges of observed monetary values it is important to recall that no significance can be attached to the specific utility units.

From these comparisons it is difficult to argue that one or the other of the two functional forms is generally superior as a smooth approximation to observed utility responses over a given range of data. The quadratic function may sometimes yield unacceptable curvature such as for grower #6A and any extensions beyond the observations would seem safer with an exponential function. However, the quadratic function may be an acceptable estimator of risk aversion over limited ranges and possesses convenient mathematical properties.

In the next section, we shall draw on these utility estimates to show how attitudes toward risk may affect contract choices and to attempt to identify areas in the contract space where alternative contract portfolios seem optimal.

VI. OPTIMAL CONTRACT PORTFOLIOS AND CONTRACT EQUILIBRIUM

In Section IV we showed how some contract portfolios could be eliminated as inefficient and identified efficient portfolios for selected sets of contract specifications. The findings are applicable regardless of the decision maker's intensity of aversion to risk. If we have additional knowledge of a contract participant's risk aversion intensity in the form of a utility function for income, we may identify the particular efficient portfolio solutions which would be optimal for that participant for any given set of contract terms. The optimal portfolio is defined as that which maximizes expected utility (see Section II). Since trade will occur only when the contracting parties agree on both sales quantities and terms, the final equilibrium solution requires further identification of the particular contract parameters which generate identical optimal sales quantities for buyer and seller. In this section the profit functions presented in Section II and the utility functions estimated in Section V are utilized to explore the contract parameter space for which various portfolio mixes are optimal, given these particular functions. An approximate final equilibrium solution is then developed.

Problem Specification

Recall that in Section II the objective functions for the contracting participants were specified by inserting expressions for expected profit and variance of profit into the expected utility functions. With total quantity to be exchanged set at predetermined levels, profit (or net margin in the case of the cooperative) may be expressed as a function of the values assigned to the contract price parameters (k, ℓ, m and n) and the proportions of sales under each of the alternative contractual arrangements or sales methods--see equations A-1, A-5, A-9 in Appendix A.[1] The expected profit functions are specified by replacing the cost and revenue terms in A-1, A-5 and A-9 with their expected values, as indicated in equations A-2, A-6, and A-10 in Appendix A. The numerical values of these terms are given in Table 3.5.[2] The variance of profit for each participant is a quadratic function of the portfolio proportions, the contract price parameters, and the variances and covariances of the price and cost values--see equations A-3, A-7 and A-11 in Appendix A. Values of these variances and covariances are given in Tables 3.5 and 3.6.

In Section II it was noted that if the decision maker's utility function is quadratic, the expected utility function may be expressed as

$$(6.1) \quad E[U(\pi)] = a + b\mu_{\pi} - c(\sigma_{\pi}^{2} + \mu_{\pi}^{2})$$

where μ_{π} is the expected value of profit, σ_{π}^{2} is variance of profit, and a, b, c are constants (b, c > 0). For the negative inverse exponential utility function, expected utility has the form

$$(6.2) \quad E[U(\pi)] = K - \theta \exp[-\lambda\mu_{\pi} + \lambda^{2}\sigma_{\pi}^{2}/2]$$

where K, θ, λ are constants > 0. The complete objective function for each participant is obtained by substituting the corresponding equations from

---

[1] S_4 is set at .75, and z at .000029 in the presentation to follow. With this specification the grower is assumed to have committed 75 percent of his tomato acreage to the cooperative on a membership basis and wishes to determine how best to sell the remaining 25 percent--i.e., at market price, cost plus, or sales-minus contract. The market price option may be interpreted as "open market" transactions or as a contract in which price is tied to market quotations. Similarly, the cooperative is assumed to contract 25 percent of its raw product needs from nonmembers. The 25 percent figure was selected to reflect the nonmember proportion that would accrue to the cooperative under study if procurement changes under consideration were enacted.

[2] Expected values for $REVC_{npst}$, REVC, $NTVCC^{a}_{pst}$, VCC_{npst}, FCC_{npst}, and NMC are omitted to preserve the confidentiality of the cooperative data.

A-2, A-6, A-10, A-3, A-7 and A-11 into (6.1) or (6.2). The optimizing problem is to choose portfolio proportions which maximize expected utility. More specifically, we obtain

(a) for the grower

$$(6.3) \qquad \text{Max } E[U(\pi_g)] = f_g(S|\mu_g, \sigma_g, k, \ell)$$

where $S_1 + S_2 + S_3 = .25$ and $S_1, S_2, S_3 \geq 0$;

(b) for the processor

$$(6.4) \qquad \text{Max } E[U(NM_p)] = f_p(V,R|\mu_p, \sigma_p, k, \ell, m, n)$$

where $V_1 + V_2 + V_3 = 1$; $R_1 + R_2 + R_3 = .25$; and $V_1, V_2, V_3, R_1, R_2, R_3 \geq 0$;

(c) for the distributor

$$(6.5) \qquad \text{Max } E[U(\pi_d)] = f_d(W|\mu_d, \sigma_d, m, n)$$

where $W_1 + W_2 + W_3 = 1$ and $W_1, W_2, W_3 \geq 0$.

In the above S, V, R, W are vectors of portfolio proportions, μ_g, μ_p and μ_d are vectors of expected values of coefficients in A-2, A-6, A-10, and σ_g, σ_p, σ_d are vectors of variances and covariances as given in equations A-3, A-7, and A-11.

In the remainder of Section VI we shall identify the approximate ranges of values of contract parameters (k, ℓ, m, n) for which various portfolio mixes appear optimal. Each set of estimates corresponds to a given participant and to particular specifications as to risk aversion and form of utility function. We shall also obtain an approximate solution for values of k, ℓ, m, n such that optimal quantities offered for sale by each contracting method are identical to optimal quantities sought for purchase by that method.

Optimal Choice Under Risk Neutrality

It is instructive to begin the analysis with solutions which specify the optimal choices of risk neutral participants. Under risk neutrality, the utility function for profit is linear and the expected utility maximizing solution is the same as the expected profit maximizing solution. The profit maximizing results are of direct interest and also provide a base for evaluating the effects of risk aversion on the optimal mix.

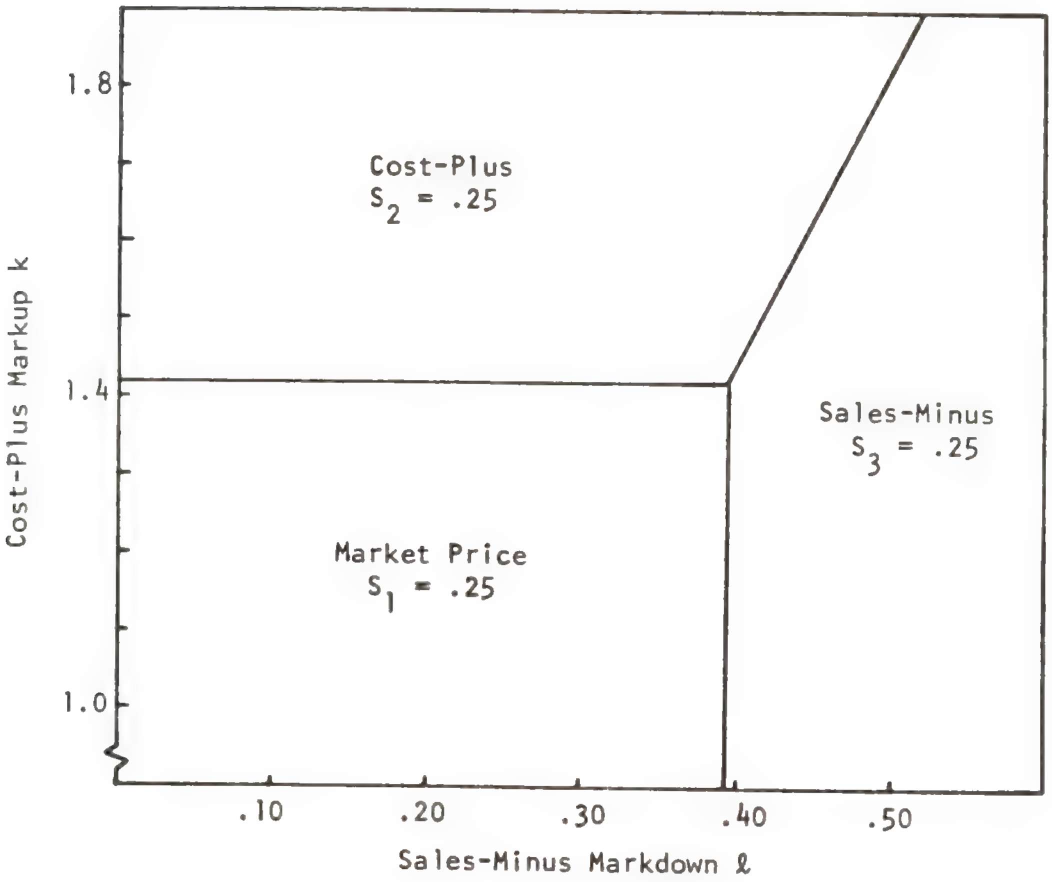
Since the expected profit function is linear (see Appendix A, equations A-2, A-6, A-10), the profit maximizing portfolio solutions always consist of a single option. If one proportion is set at 1.0 (or .25 in the specification of the grower or cooperative purchase problems) and the others at zero, the expected profit equations may be solved for the range of values of contract price parameters for which each option gives the largest expected profits.

Optimal choice regions for a grower with 1,210 acres of tomatoes and 1,940 acres of corn, wheat, and beans are graphed in Figure 6.1. The market price and cost expectations are such that if all sales were made at market prices, expected returns for tomato production would be 42 percent over cash costs and 22 percent over total costs. Thus, a cost-plus markup of $k = 1.42$ would yield the same expected profit as the market price alternative. The value of ℓ by which market price and sales-minus yield the same expected profit is .396 (see Table 3.5).[1/]

[1/] This is calculated by dividing expected per acre tomato market value (MV^a_{tom}) by expected per acre paste market value (MV^a_{pst}). The grower sales-minus option, like the coop sales-minus purchase option, refers only to coop paste revenue and assumes all paste has been sold at the market price. This assumption is required to preserve the linear character of grower and cooperative profit. See Section II.

FIGURE 6.1

Maximum Expected Profit Choices for a Grower with 75 Percent of Production Allocated to the Cooperative on a Profit-Share Basis, the Remainder Either Cost-Plus, Sales-Minus or Market Price. AAF = 1210, OAFF = 1940



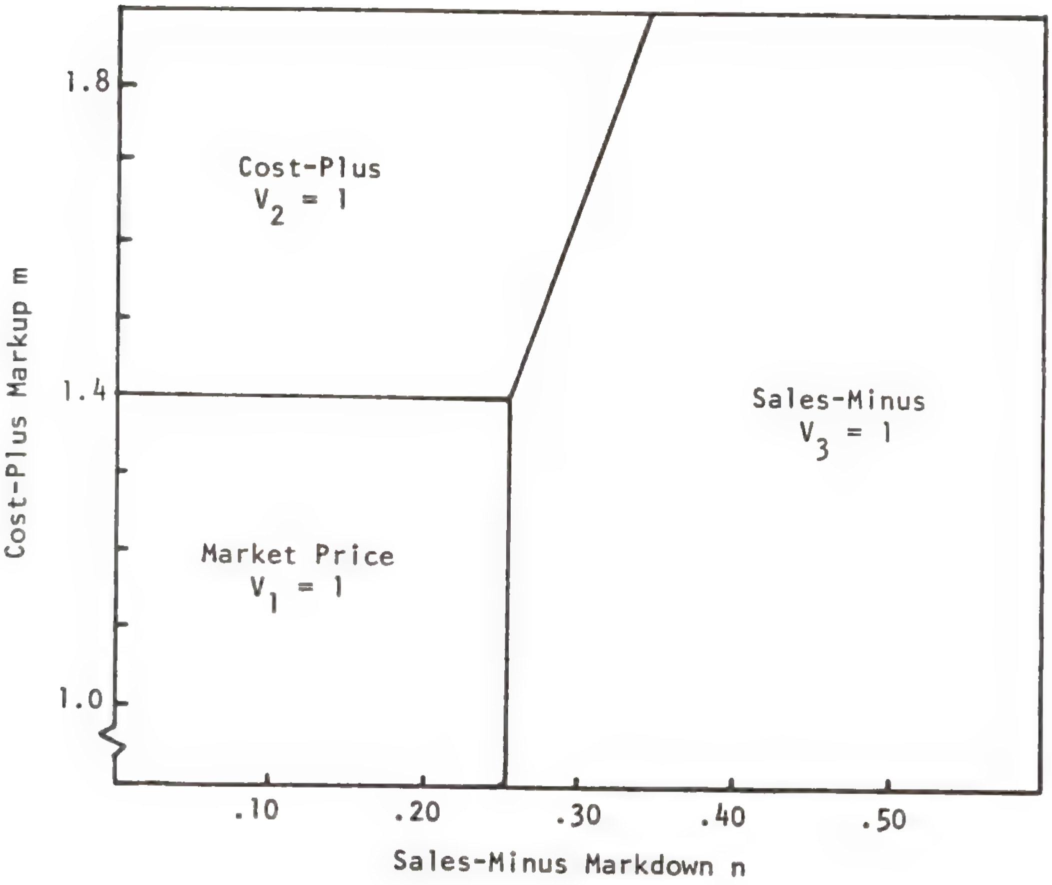
The determination of highest expected return regions for each cooperative sales option is slightly more complex than for the grower curve because there are two sets of proportions, V and R, and four contract parameters, m, n, k, ℓ. Alternately assigning 1.0 to each V_i and zeros to the remaining V_i provides a set of linear functions of m and n which may be solved to define regions of highest return for each sales contract alternative. A similar process may be used to define maximum expected return regions for each purchase alternative.

Figure 6.2 shows the range of values of m and n for which each sales option gives highest expected return with production at 75,300 tons of bulk packed 32 percent tomato paste (18,860 acre equivalents) and k and ℓ set at 1.42 and .396. It will be recalled that at those levels of k and ℓ, the expected payment received by growers is identical for each alternative, i.e., market price, cost-plus, or sales-minus (see Figure 6.1). Cooperative price and cost expectations are such that if all purchases and sales were made at market prices, rate of profit over cash costs would be 40 percent and rate of profit over total costs 15 percent. Thus, a cost-plus markup of m = 1.4 represents equal profitability between market price and cost-plus options.[1/] Similarly, the market price and sales-minus options provide equal expected profitability at n = .2534. The latter value is computed by dividing expected per acre market value of tomato paste (MV^a_{pst}) by expected per acre market value of tomato sauce (MV^a_{sce})--see Table 3.5.

[1/] The cooperative E-V frontiers presented in Section IV were constructed under an expectation of lower nontomato cash costs and have equal profit for market price and cost-plus at m = 1.65. Subsequent discussions with a representative of the cooperative indicated that cost-plus markups as high as 1.65 were not currently realistic. This suggested that our nontomato cash costs were too low. Increasing these costs lowered the break-even value of cost-plus with market price to m = 1.4. The E-V frontiers were not recalculated since the general conclusions are not greatly altered and the computational cost is fairly large.

FIGURE 6.2

Maximum Expected Net Revenue Contract Choices for the Cooperative Assuming the Grower Cost-Plus Markup Is $k = 1.42$, the Sales-Minus Markdown ℓ Is .396 and ACC = 18,860



The distributor is assumed to contract for all of the cooperative sales of 32 percent tomato paste. Paste is reprocessed into various sauces and sold to food retailers and others. The purchase options are market price, cost-plus, and sales-minus. The expected profit function is specified as in A-10. The range of values of m and n for which each purchase option gives maximum expected profit to the distributor is graphed in Figure 6.3.

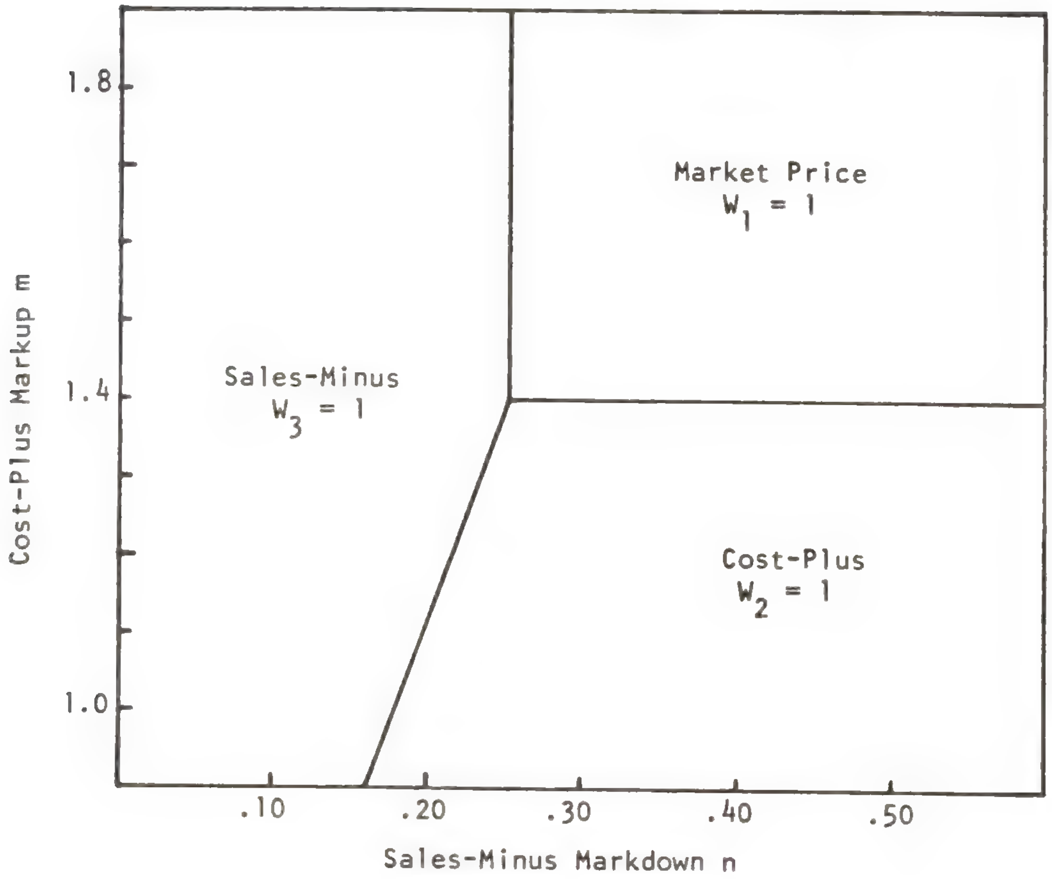
If Figure 6.2 is laid on Figure 6.3, it is evident that in areas where the distributor prefers purchases at market price, the cooperative prefers sales at cost-plus or sales-minus. If the distributor behaves as a pure profit maximizer and dominates the bargaining process, the cooperative can not expect to enter into any cost-plus or sales-minus contracts where $m > 1.4$ or $n > .2534$. Conversely, if the cooperative behaves as a pure profit maximizer (risk neutral) and dominates the bargaining process, it would not enter into any cost-plus or sales-minus contracts where $m < 1.4$ or $n < .2534$. If $m = 1.4$ or $n = .2534$, the latter contracts could be employed, but this would offer no advantage to either party over market price. However, if either party is risk averse, cost-plus or sales-minus contracting may occur for values of m and n other than 1.4 and .2534 and the optimal solutions may provide a mix of contract options.

Optimal Choice Under Risk Aversion

Our limited survey of growers and cooperative decision makers suggests that they may vary widely in their attitudes toward risk. Although most decision makers exhibited characteristics of risk aversion, a few may be risk takers, preferring situations with lower expected pay-offs if they offer possible opportunities for larger gains. Many others appear to be risk neutral. Still others may be risk takers for some levels of income and then risk neutral or risk averse as income varies. With so much

FIGURE 6.3

Maximum Expected Profit Choices for the Distributor



variation, optimal contract choices cannot be specified in any general sense, but may vary with the proclivities of the individual participants. And in many cases, equilibrium choices of contractual terms and portfolios as determined by (say) management personnel may be considered less than optimal by others in the organizations, and may in fact vary with changes in authority. In view of this, the explorations of contract parameter space and the specific equilibrium solutions which follow must be regarded as illustrative rather than precise guidelines. However, the results provide a basis for some generalizations about the effects of risk aversion and suggest some interesting conclusions about the establishment of contract terms.

Selection of Representative Utility Functions

Since the objective in this section is to show how attitudes toward risk may affect the optimal contract mix, we have selected utility functions and operational points on these curves that reflect moderate to strong degrees of risk aversion, consistent with the observed responses. We have also made limited comparisons between results obtained with quadratic and exponential functional forms.

To represent the grower component of the model we selected the utility function for Grower 9 in Table 5.4, $U = 50.76 + 30.93M - 4.69M^2$ (M in \$1,000). It was discovered in the initial optimization calculations that for the acreage levels actually observed for Grower 9 (Table 5.1), his expected profit implied a degree of risk aversion such that his optimal choice was always that of a profit maximizer as indicated in Table 6.1. Since it was of interest to see how his portfolio mix might vary under more risk averse circumstances, his acreages were increased to 1,210 acres of tomatoes and 1,940 acres of corn, wheat, and beans. This places him at approximately the

\$250,000 point (in terms of equivalent pre-tax annual returns) on his utility function, still in the positively sloped portion. Inspection of Figure 5.1 and the placement of the higher observations suggests this may reasonably approximate the grower's risk aversion in this profit range.

To represent the cooperative attitude toward risk, we selected the utility functions estimated for the cooperative executive. Solutions were obtained for both the quadratic utility function ($U = 59.8 + 3.1M - .06M^2$) and the exponential form ($U = 120 - 59 \exp[-.044M]$), where M is expressed in millions of dollars. Portfolio solutions were examined for 75,800 tons of paste sold, as in Figure 6.2. Solutions for substantially lower levels (e.g., 45,480 tons) were associated with lower levels of risk aversion and resulted in essentially the profit maximizing solution for both types of utility functions. In the case of the exponential utility function this condition prevailed even at the higher level of quantity and net revenue; that is, the cooperative executive always selected the single contracting option which gave highest profit for given parameters, rather than some mix of contracts. This was surprising since the quadratic function (which did produce a portfolio mix) and the exponential function appear similar when graphed. However, comparison of the absolute risk aversion coefficients at 75,800 tons (\$21 million net margin) shows a coefficient of about .21 for the quadratic function compared to only .044 for the exponential.

In order to evaluate the behavior of offer curves for exponential utility under more risk averse conditions, we constructed a hypothetical function with an arbitrarily selected value of $\lambda = .45$. This function gave portfolio solutions similar in their relation to contract price parameters to the findings obtained with the quadratic function in its higher range of risk aversion. In the analysis to follow we shall present only the quadratic utility function results.

It was noted earlier that we were unable to obtain interviews with representatives of the reprocessing firm (distributor) which contracted with the cooperative for tomato paste. In order to complete our system, we have specified a hypothetical distributor utility function which enables us to determine an illustrative equilibrium solution. This function, $U = 1M - .015M^2$ (M in millions of dollars), was designed to cover a somewhat wider dollar range than the cooperative utilities in order to reflect the larger volume of many corporate distributors. The function is also less risk averse than the cooperative executive's utility, under the assumption that farmer cooperatives may be more security conscious than their corporate counterparts. The distributor function reaches a maximum at \$33.33 million compared to the \$25.3 million of the cooperative executive. The former has a coefficient of absolute risk aversion of .048 at the \$12.5 million point, and the latter a coefficient of .075 at this point.

Equivalent Annual Rates of Present Value Profit Sums

The utility functions developed in the previous section are expressed in terms of annual profit rates whereas the expected profit for contracting decisions is a ten-year discounted sum of profits. For computational purposes it is necessary either to convert the discounted sum of profits to an equivalent annual rate or to convert utility functions to reflect present value sums. We shall follow the former procedure.

Let $E(\pi_T)$ be the ten-year discounted sum of expected annual profits, $E(\pi_t)$ the profits expected to be earned in year t, and π_y a constant expected annual rate of profit. π_y is defined in relation to the variable expected series $E(\pi_t)$ by

$$E(\pi_T) = \sum_{i=1}^{10} \frac{E(\pi_t)}{(1+i)^t} = \sum_{i=1}^{10} \frac{\pi_y}{(1+i)^t} \,.$$

Thus,

$$\pi_y = \frac{E(\pi_T)}{h} \qquad \text{where } h = \sum_{i=1}^{10} \frac{1}{(1+i)^t} .$$

For $i = .02$, $h = 8.98$, and the present value sums of expected profit over the ten-year planning period are converted to annual equivalents by dividing by 8.98. It also follows that constant annual profit variance π_z is defined by

$$\pi_z = \mathrm{var}(\pi_T)/h^2 = \sum_{t=1}^{T} \left[\frac{\mathrm{var}(\pi_t)}{(1+i)^{2t}}\right] h^2 ,$$

so that present value sums of profit variances are converted to an annual rate by dividing by $(8.98)^2 = 80.6404$.

Relation of Portfolio Shares to Contract Parameters

With the individual utility functions specified as above, we may solve readily for the particular sales portfolio that maximizes expected utility for any given set of contract specifications. Our objective now is to show how these portfolio shares vary in relation to the contract parameter values, k, ℓ, m and n.

The problem may be solved mathematically by reference to the first order conditions for maximizing the expected utility functions 6.3, 6.4 and 6.5. The partial derivatives of these functions with respect to the portfolio proportions provide a set of equations that are linear in terms of the portfolio proportions and quadratic in terms of the contract parameters (see Appendix B). These equations may be solved for the optimal portfolio shares, with the solutions depending on the values assigned to the contract parameters. Since portfolio shares are contained between zero and one (or zero and .25 for the grower and for cooperative purchases), both boundary and interior solutions are required. For the grower's three sales options there are seven types of solutions:

(1) $0 < S_1, S_2, S_3 < .25$

(2) $S_1 = 0, 0 < S_2, S_3 < .25$

(3) $S_2 = 0, 0 < S_1, S_3 < .25$

(4) $S_3 = 0, 0 < S_1, S_2 < .25$

(5) $S_1 = .25, S_2, S_3 = 0$

(6) $S_2 = .25, S_1, S_3 = 0$

(7) $S_3 = .25, S_1, S_2 = 0.$

A similar set of solutions is required for the distributor's three options but with the S's replaced by W's and .25 replaced by 1.0. The cooperative processor solution set is more complex because of the existence of interdependent sales and purchase alternatives. For each type of cooperative sales solution there are seven possible types of purchase solutions, giving a total of 49 variations. Examination of the empirical solutions (to be discussed shortly) suggested, however, that the range of parameter values for which there is interdependence is fairly narrow. Thus, the processor purchase and sales portfolio decisions may be treated as independent over much of the range of parameter variation with relatively little loss in optimizing accuracy. With this simplification the cooperative purchase and sales solution requirements are similar in format to the grower model.

The boundary and interior solutions which express optimal grower sales options as functions of the contract price parameters k and ℓ are developed in Appendix B. Most of the solution equations are quite complex. For example, the interior solutions involving a mix of all three options have fourth degree polynomial terms in both numerator and denominator. Such functions are very difficult to portray individually and the set of functions is virtually impossible to solve for the equilibrium values of parameters under which exchange occurs.

To cope with these difficulties we developed simplified mathematical approximations of each of the solution equations. This was accomplished by varying the values of k, ℓ, m, and n and obtaining quadratic programming solutions of portfolio shares for each set of parameter values [Cutler and Pass]. The observations generated by this process were then used to obtain linear approximations of the boundary and interior solution equations.

Optimal portfolio solutions using the quadratic utility functions described earlier in this section are given in Tables 6.1, 6.2, and 6.3. Linear approximations to the several solution equations, based on ordinary least squares fits to the data subsets, are given in Tables 6.4, 6.5, and 6.6. Although the standard errors of these equations have no direct statistical interpretation, the high R^2 values for most equations suggest that the linear equations provide reasonably good approximations of the more complex functions developed in Appendix B for the range of observations. Note that although seven types of border solutions are involved for the three-option case, only four equation approximations are required. The solution specifications for the three cases where a particular option is employed 100 percent (e.g., $S_1 = .25$ or $S_2 = .25$ or $S_3 = .25$) may be derived from the partial border solutions where one option is zero and the other two greater than zero are less than the maximum value.

Because of the very narrow range of parameter values which yield interior solutions for the cooperative purchase portfolio, it turned out that the quadratic programming solutions did not provide all the observations needed to generate the complete set of cooperative solution equations. Solution equations for R_1 and R_3 with $R_2 = 0$, and for R_1 and R_2 with $R_3 = 0$, are given in Part 2 of Table 6.5. Only two observations were obtained for $R_1 = 0$ and R_2 and R_3 not zero, and only one set of assigned parameter values (observation 26) resulted in a mix of all three purchase options. In the final equilibrium exchange solutions, the optimal purchase proportions will be evaluated by interpolation from the data set rather than direct equation solutions. Part 3 of Table 6.5 gives an indication

TABLE 6.1

Observations on Grower's Contract Portfolio Offer Curves for Sale of Tomatoes, Nonmember Options Only[a]

Solution number	Sales contract parameters		Sales contract options		
	Cost plus markup (k)	Sales-minus markdown (ℓ)	Market price (S_1)	Cost plus (S_2)	Sales-minus (S_3)
			(proportions of portfolio)		
1	1.400	.400	0	.188	.062
2	1.400	.350	0	.250	0
3	1.375	.350	.042	.208	0
4	1.350	.400	0	.077	.173
5	1.350	.383	.109	.141	0
6	1.350	.375	.109	.141	0
7	1.350	.350	.109	.141	0
8	1.325	.400	0	.042	.207
9	1.325	.383	.134	.063	.052
10	1.325	.380	.158	.086	.006
11	1.325	.375	.160	.090	0
12	1.300	.400	0	.020	.230
13	1.300	.383	.140	.008	.101
14	1.300	.375	.197	.053	0
15	1.300	.350	.197	.053	0
16	1.275	.400	.009	.004	.236
17	1.275	.383	.140	0	.110
18	1.275	.375	.199	.004	.046
19	1.275	.350	.222	.028	0

[a] Total tomato acreage contracted is 1,210 acres. Of this 75 percent or 908 acres are sold on a membership basis. Utility is $U = 50.76 + 3.098M - .0469M^2$ (Grower #9).

TABLE 6.2

Observations on Cooperative Contract Portfolio Offer Curves for Tomato Paste Sales and Tomato Purchases, Assuming Quadratic Money Utility[a/]

Solution number		Sales contract parameters		Purchase contract parameters		Sales contract proportions			Purchase contract proportions		
		Cost-plus markup (m)	Sales-minus markdown (n)	Cost-plus markup (k)	Sales-minus markdown (ℓ)	Market price (V_1)	Cost-plus (V_2)	Sales-minus (V_3)	Market price (R_1)	Cost-plus (R_2)	Sales-minus (R_3)
A. m, n varying, k, ℓ constant	1	1.41	.248	1.3500	.3800	.056	.943	0	0	0	.250
	2	1.41	.250	1.3500	.3800	0	.874	.126	0	0	.250
	3	1.41	.252	1.3500	.3800	0	.743	.257	0	0	.250
	4	1.41	.254	1.3500	.3800	0	.607	.393	0	0	.250
	5	1.40	.246	1.3500	.3800	.157	.843	0	0	0	.250
	6	1.40	.248	1.3500	.3800	.157	.843	0	0	0	.250
	7	1.40	.250	1.3500	.3800	0	.794	.206	0	0	.250
	8	1.40	.254	1.3500	.3800	0	.488	.512	0	0	.250
	9	1.39	.246	1.3500	.3800	.283	.717	0	0	0	.250
	10	1.39	.248	1.3500	.3800	.268	.697	.035	0	0	.250
	11	1.39	.250	1.3500	.3800	.173	.582	.245	0	0	.250
	12	1.39	.252	1.3500	.3800	.064	.468	.468	0	0	.250
	13	1.39	.254	1.3500	.3800	0	.354	.646	0	0	.250
	14	1.38	.246	1.3500	.3800	.421	.578	0	0	0	.250
	15	1.38	.248	1.3500	.3800	.375	.511	.113	0	0	.250
	16	1.38	.250	1.3500	.3800	.270	.387	.344	0	0	.250
	17	1.38	.252	1.3500	.3800	.150	.278	.572	0	0	.250
	18	1.38	.254	1.3500	.3800	.034	.203	.763	0	0	.250
	19	1.37	.246	1.3500	.3800	.562	.438	0	0	0	.250
	20	1.37	.248	1.3500	.3800	.473	.309	.218	0	0	.250
	21	1.37	.250	1.3500	.3800	.356	.188	.456	0	0	.250
	22	1.37	.252	1.3500	.3800	.226	.098	.676	0	0	.250
	23	1.36	.246	1.3500	.3800	.655	.242	.103	0	0	.250
	24	1.36	.248	1.3500	.3800	.552	.111	.338	0	0	.250
	25	1.35	.254	1.3500	.3800	.125	0	.875	0	0	.250
B. Full inter-action	26	1.38	.250	1.3363	.3975	.223	.433	.345	.068	.114	.068
	27	1.38	.248	1.3363	.3975	.332	.618	.050	.250	0	0
	28	1.37	.250	1.3363	.3975	.382	.164	.450	0	0	.250
	29	1.37	.248	1.3363	.3975	.508	.292	.200	0	0	.250
	30	1.38	.250	1.3350	.3975	.178	.438	.384	0	.250	0
	31	1.38	.248	1.3350	.3975	.332	.618	.050	.250	0	0
	32	1.37	.250	1.3350	.3975	.386	.164	.450	0	0	.250
	33	1.37	.248	1.3350	.3975	.508	.292	.200	0	0	.250
	34	1.38	.250	1.3363	.4000	.192	.452	.356	.076	.164	0
	35	1.38	.248	1.3363	.4000	.332	.618	.050	.250	0	0
	36	1.37	.250	1.3363	.4000	.271	.228	.502	0	.250	0
	37	1.37	.248	1.3363	.4000	.449	.387	.164	.219	0	.031
	38	1.38	.250	1.3350	.4000	.178	.438	.384	0	.250	0
	39	1.38	.248	1.3350	.4000	.332	.618	.050	.250	0	0
	40	1.37	.250	1.3350	.4000	.271	.228	.502	0	.250	0
	41	1.37	.248	1.3350	.4000	.449	.387	.164	.219	0	.031
C. m, n constant, k, ℓ varying	42	1.38	.250	1.3570	.3980	.223	.484	.293	.250	0	0
	43	1.38	.250	1.3560	.3970	.295	.374	.331	0	0	.250
	44	1.38	.250	1.3370	.3975	.279	.399	.322	.058	0	.192
	45	1.38	.250	1.3365	.3975	.279	.399	.322	.058	0	.192
	46	1.38	.250	1.3365	.3974	.294	.376	.330	.005	0	.245
	47	1.38	.250	1.3363	.3974	.294	.376	.330	.005	0	.245
	48	1.38	.250	1.3361	.3975	.187	.447	.366	.050	.200	0
	49	1.38	.250	1.3360	.3975	.185	.445	.370	.036	.214	0
	50	1.38	.250	1.3350	.3975	.178	.438	.384	0	.250	0

[a/] Total sales are 75,800 tons 32 percent paste. Utility is $59.8 + 3.1M - .06M^2$.

TABLE 6.3

Observations on Distributor Contract Portfolio Offer Curves for Tomato Paste Purchases[a]

	Purchase contract parameters		Purchase contract proportions		
Solution number	Cost-plus markup (m)	Sales-minus markdown (n)	Market price (W_1)	Cost-plus (W_2)	Sales-minus (W_3)
1	1.3800	.255	.185	0	.815
2	1.3800	.256	.467	0	.533
3	1.3800	.257	.740	.002	.258
4	1.3800	.258	.884	.116	0
5	1.3700	.255	.071	.234	.695
6	1.3700	.256	.261	.395	.344
7	1.3700	.257	.446	.526	.028
8	1.3700	.258	.463	.537	0
9	1.3675	.255	.007	.364	.629
10	1.3600	.254	0	.420	.580
11	1.3600	.255	0	.642	.358
12	1.3600	.256	.018	.831	.151
13	1.3600	.257	.115	.885	0
14	1.3500	.254	0	.756	.244
15	1.3500	.255	0	.931	.069

[a] Total purchases are 75,800 tons of 32 percent paste. Utility is $U = 1M - .015M^2$ (in millions of dollars).

TABLE 6.4

OLS Approximations of Grower Offer Curves[a]

	Dependent variable	Constant term	k	ℓ	Number of observations	R^2	Boundary lines for k and ℓ
(1) Interior solution $(0 < S_1, S_2, S_3 < .25)$	S_1	3.5167	-.3087 (.0147)	-7.7811 (.5263)	6[b]	.987	$k = 11.3920 - 25.2060\ell$ $k = 1.3354 - .1455\ell$ $k = -.6172 + 5.1464\ell$
	S_2	-2.3644	1.7706 (.3616)	.2576 (1.2980)	6[b]	.894	
	S_3	-.9023	-1.4619	7.5235			
(2) $S_1 = 0, \quad 0 < S_2, S_3 < .25$ [(2)-(a) Appendix B]	S_2	-1.2565	1.4778 (.2196)	-1.6070 (.5557)	6[c]	.973	$k = 11.3920 - 25.2606\ell$ $k = .8503 + 1.0874\ell$ $k = 1.0194 + 1.0874\ell$
	S_3	1.5065	-1.4778	+1.6070			
(3) $S_2 = 0, \quad 0 < S_1, S_3 < .25$ [(2)-(b) Appendix B]	S_1	3.0560		-7.6166 (.0797)	3[d]	.999	$.368 < \ell < .4012$ $k = 1.3354 - .1455\ell$
	S_3	-2.8060		+7.6166			
(4) $S_3 = 0, \quad 0 < S_1, S_2 < .25$ [(2)-(c) Appendix B]	S_1	2.5204	-1.7920 (.1915)		5[e]	.967	$1.2670 < k < 1.4065$ $k = -.6172 + 5.1464\ell$ $k = 1.0194 + 1.0874\ell$
	S_2	-2.2704	1.7920				
(5) $S_1 = .25, \quad S_2, S_3 = 0$							$k \leq 1.267$ $\ell \leq .368$
(6) $S_2 = .25, \quad S_1, S_3 = 0$							$k \geq 1.406$ $k = 1.0194 + 1.0874\ell$
(7) $S_3 = .25, \quad S_1, S_2 = 0$							$k = .8503 + 1.0874\ell$ $\ell \geq .4012$

[a] In the above standard errors (in parentheses) have no statistical significance. They are cited only to give some indication of goodness of fit.

[b] Observations 5, 9, 10, 13, 16, 18 in Table 6.1.

[c] Observations 1, 2, 4, 8, 12, 16 in Table 6.1.

[d] Observations 16, 17, 18 in Table 6.1.

[e] Observations 3, 5, 11, 15, 19 in Table 6.1.

TABLE 6.5

OLS Approximations of Processor Offer Curves[a]

1. Sales side

	Dependent variable	Constant term	m	n	R^2	Number of observations	Boundary lines for m and n
(1) Interior solutions $(0 < V_1, V_2, V_3 < 1)$ $k = 1.35, \ell = .38$	V_1	26.5474	-8.9486 (.2775)	-55.7513 (1.2786)	.998	12[a]	$m = 2.9667 - 6.2302n$
	V_2	-12.4668	19.0460 (.3893)	-53.6908 (1.7937)	.996	12[b]	$m = +.6546 + 2.819n$
	V_3	-13.0806	-10.0974	109.4421			$m = -1.292 + 10.8246n$
(2) $V_1 = 0, \quad 0 < V_2, V_3 < 1$ $k = 1.35, \ell = .38$	V_2	2.7517	11.6488 (1.0116)	-73.1395 (4.2005)	.995	6[c]	$m = 2.8005 - 5.5843n$ $m = -.1504 + 6.2787n$ $m = -.2362 + 6.2787n$
	V_3	-1.7517	-11.6488	73.1395			
(3) $V_2 = 0, \quad 0 < V_1, V_3 < 1$ $k = 1.35, \ell = .38$	V_1	17.7145		-69.3000 (4.3148)	.992	4[d]	$.2416 < n < .2556$
	V_3	-16.7145		69.3000			$m = .5832 + 3.1081n$
(4) $V_3 = 0, \quad 0 < V_1, V_2 < 1$ $k = 1.35, \ell = .38$	V_1	18.1502	-12.8462 (.4160)		.996	6[e]	$1.3350 < m < 1.4130$
	V_2	-17.1502	12.8462				$m = -1.2739 + 10.7355n$
(5) $V_1 = 1, \quad V_2, V_3 = 0$							$m \leq 1.335$ $n \leq .2416$
(6) $V_2 = 1, \quad V_1, V_3 = 0$							$m \geq 1.4130$ $m = -.1504 + 6.2787n$
(7) $V_3 = 1, \quad V_2, V_3 = 0$							$m = -.2362 + 6.2787n$ $n \geq .2556$

(Table 6.5 continued)

Table 6.5 continued

2. Purchase side

	Dependent variable	Constant term	k	ℓ	R^2	Number of observations	Boundary lines for m and n
(a) $R_3 = 0$, $0 < R_1, R_2 < .25$ $m = 1.38$, $n = .25$	R_1	-177.4560	132.8530 (2.5737)		.999	3[f]	
	R_2	177.7060	-132.8530				
(b) $R_2 = 0$, $0 < R_1, R_3 < .25$ $m = 1.38$, $n = .25$	R_1	-29.4023		74.0554 (15.4845)	.958	3[g]	
	R_3	29.6523		-74.0554			

3. Interaction

	Dependent variable	Constant term	m	n	R^2	Number of observations	Boundary lines for m and n
$0 \leq V_1, V_2, V_3 \leq 1$ $0 \leq R_1, R_2, R_3 \leq .25$ $1.3350 \leq k \leq 1.3363$	R_1	18.2093	4.9500 (1.8500)	-100.2500 (9.2499)	.992	4[h]	
$.3975 \leq \ell \leq .4000$	R_2	-13.2180	-6.8000 (6.8000)	91.0000 (34.0000)	.891	4[h]	

---

a/ The standard errors (in parentheses) have no statistical significance. They are cited only to give an indication of the goodness of fit.

b/ Observations 10, 11, 12, 15, 16, 17, 18, 20, 21, 22, 23, 24 in Table 6.2.

c/ Observations 2, 3, 4, 7, 8, 13 in Table 6.2.

d/ Observations 22, 23, 24, 25 in Table 6.2.

e/ Observations 1, 5, 6, 9, 14, 19 in Table 6.2.

f/ Observations 34, 48, 49 in Table 6.2.

g/ Observations 41, 44, 46 in Table 6.2.

h/ Observations 26, 27, 36, 41 in Table 6.2.

TABLE 6.6

OLS Approximations of Distributor Offer Curves[a]

	Dependent variable	Constant term	m	n	R^2	Number of observations	Boundary lines for m and n
(1) Interior solution ($0 < W_1, W_2, W_3 < 1$)	W_1	-83.2702	25.9340 (1.0826)	187.5000 (1.4134)	.999	5[b]	$m = 3.2109 - 7.2299n$
	W_2	31.5080	-49.9991 (6.6380)	145.9990 (8.6664)	.997	5[b]	$m = .6302 + 2.9200n$
	W_3	52.7622	24.0651	-333.4990			$m = -2.1925 + 13.8581n$
(2) $W_1 = 0, \quad 0 < W_2, W_3 < 1$	W_2	-4.4028	-32.5510 (1.4713)	193.2960 (20.1472)	.996	5[e]	$m = 3.2109 - 7.2299n$ $m = -.1574 + 6.0263n$
	W_3	5.4026	32.5510	-193.2960			$m = -.1660 + 5.9383n$
(3) $W_2 = 0, \quad 0 < W_1, W_3 < 1$	W_1	-70.5760		277.5000 (2.6033)	.999	3[d]	$.2543 \leq n \leq .2580$
	W_3	71.5760		-277.5000			$m = .6302 + 2.9200n$
(4) $W_3 = 0, \quad 0 < W_1, W_2 < 1$	W_1	-52.1892	38.4500 (2.1073)		.997	3[c]	$1.357 \leq m \leq 1.383$
	W_2	53.1892	-38.4500				$m = -2.1925 + 13.8581n$
(5) $W_1 = 1, \quad W_2, W_3 = 0$							$m \geq 1.3830$ $n \geq .258$
(6) $W_2 = 1, \quad W_1, W_3 = 0$							$m \leq 1.3570$ $m = -.1660 + 5.9383n$
(7) $W_3 = 1, \quad W_1, W_2 = 0$							$m = -.1574 + 6.0263n$ $n < .2543$

[a] In the above the standard errors (in parentheses) have no statistical significance. They are cited only to give some indication of goodness of fit.

[b] Observations 5, 6, 7, 9, 12 in Table 6.3.

[c] Observations 4, 8, 13 in Table 6.3.

[d] Observations 1, 2, 3 in Table 6.3.

[e] Observations 9, 10, 11, 14, 15 in Table 6.3.

of the effect of changes in sales contract parameters on purchase portfolios. Note that the equations are applicable only over very narrow ranges of k and ℓ.

The equations presented in Tables 6.4, 6.5, and 6.6 are referred to as "offer curves." They show how the quantities (proportions) offered for sale or purchase by each contract method vary in relation to the contract price parameters. In this sense they have attributes of supply and demand curves. Quantities offered for sale by the processor by cost-plus contract, for example, increase with increases in the cost-plus parameter m and decrease with increases in the sales-minus parameter n. Sales at market price decrease with increases of both m and n. In this model the market price expectation remains constant, but if it were to increase it would lead to increases in sales by that method.

In the case of sales by the grower, the positive sign for the coefficient of ℓ in the interior solution equation for S_2 (set (1) Table 6.4) was unexpected. It is explained by the fact that grower variable costs and cooperative market paste revenue ($VCF^{a,o}_{tom}$ and MV^{a}_{pst}) are slightly negatively correlated and negative correlation can induce complementarity of contract options if risk aversion is strong enough.

On the demand side all coefficients have the signs that would be expected a priori. For example, the optimal quantity of cost-plus contracts for the distributor decreases with increases in the cost-plus parameter m and increases with increases in n.

By setting the portfolio shares at zero or one (or .25 for grower sales or cooperative purchases) in the various offer curves we obtain equations which approximately define the borders of the regions in contract parameter space where each option is used exclusively, not at all, or in some mix with other options. These calculated border equations are given in the right hand side of Tables 6.4, 6.5 and 6.6 and are graphed in Figures 6.4, 6.5 and 6.6 for the grower sales alternatives, the cooperative sales alternatives and the distributor purchase alternatives. Because of the approximate nature of the various offer curve solutions, these calculated border lines (the solid lines in Figures 6.4, 6.5, 6.6) do not divide the parameter space unambiguously. That is, there are areas near the corners where the type of solution mix is not clearly defined. The dashed lines represent small arbitrary adjustments in selected borderlines (but consistent with the basic data set) which remove the ambiguities in the parameter space. If the observations in Tables 6.1, 6.2, and 6.3 were plotted on these diagrams, most would fall in the proper region in price parameter space and the few that do not would be close to the edge of the appropriate area. This suggests that linear fits to the offer curve sets perform reasonably well as approximations to these functions.

A comparison of the utility maximization outcomes of Figures 6.4, 6.5 and 6.6 with the profit maximization outcomes of Figures 6.1, 6.2, and 6.3 brings out the impact of increased risk aversion and reveals that the exchange problem has been significantly transformed. Under profit maximization the price parameter space is divided into regions in which each sales or purchase option is used exclusively, and cost-plus or sales-minus contracting occurs only at parameter values such that traders are indifferent between these contracts and the market price option. Under utility maximization there are regions in the parameter space where a mix of sales options is preferred by all participants. Furthermore, cost-plus or sales-minus contracts may appear within an optimal solution mix for values of m and n or k and ℓ for which the market price option gives the highest expected

FIGURE 6.4

Regions of Optimal Contract Portfolios for the Grower Sales Alternatives

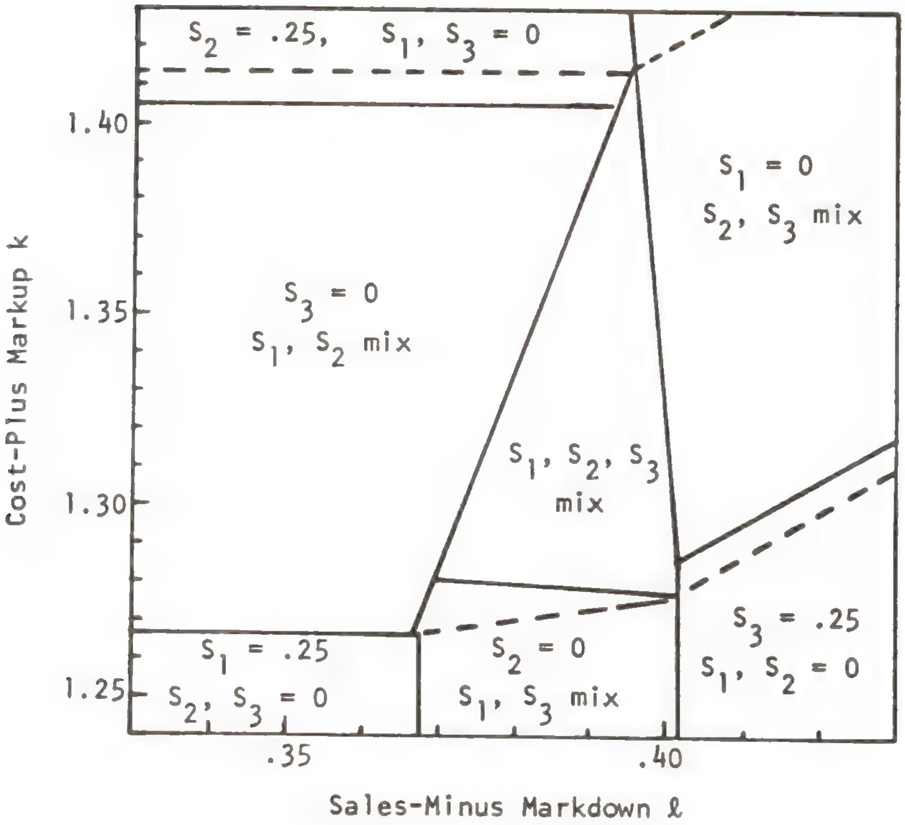


FIGURE 6.5

Regions of Optimal Contract Portfolios
for the Cooperative Sales Alternatives
($1.335 \leq k \leq 1.35$, $.38 \leq \ell \leq .400$)

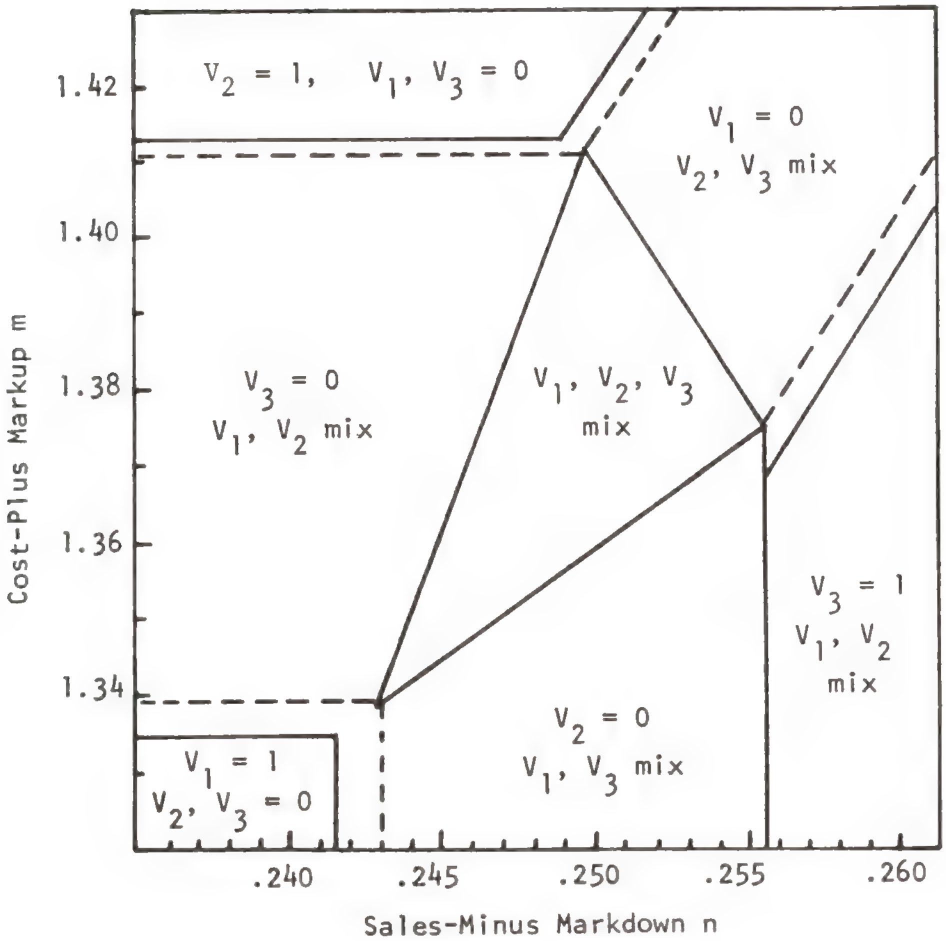
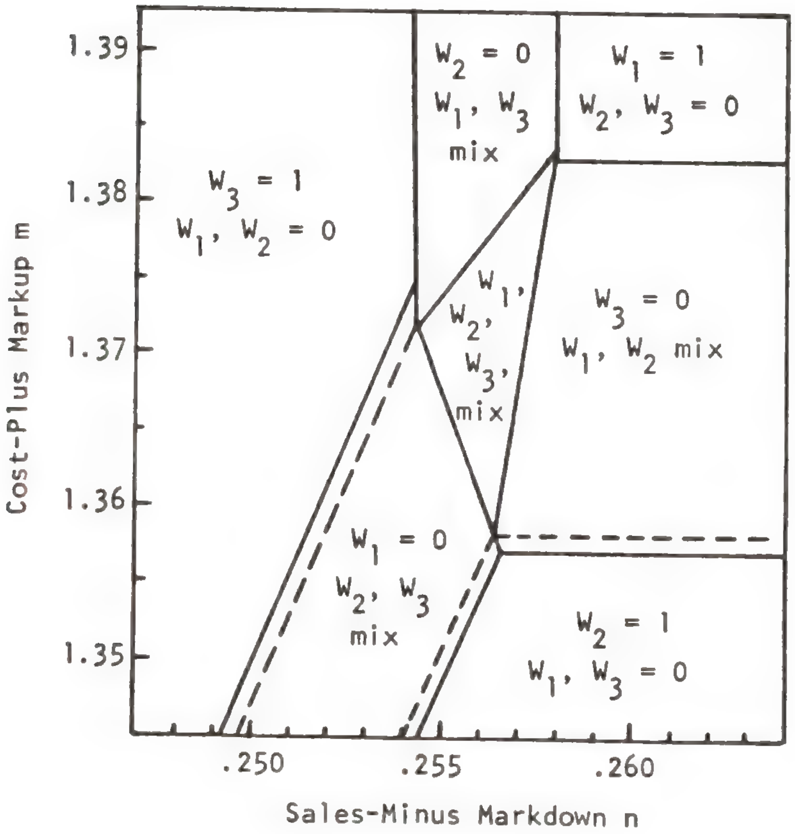


FIGURE 6.6

Regions of Optimal Contract Portfolios for the Distributor Sales Alternatives



profit. As the degree of risk aversion increases so do the regions within which a mix, rather than a single option, is optimal.

Contract Equilibrium

While the set of offer curves given in Tables 6.4, 6.5 and 6.6 predict the optimal portfolio shares for each participant for any given set of contract parameters (given specific utility functions and cost and price expectations and variances), trade occurs only for parameter values which equate sales quantities desired by both buyer and seller. As was noted earlier, the equilibrium solution requires simultaneous determination of the values of m, n, k and ℓ since optimal cooperative sales and purchase portfolios are interdependent. A complete specification of offer curves to cover all interior and bordered solutions for the cooperative would require consideration of 49 possible solution sets (see previous discussion pertaining to Appendix B). To simplify the equilibrium solutions we first solve separately for the values of m and n which equilibrate the cooperative and distributor sales and purchase portfolios, with k and ℓ set at 1.35 and .38. We then solve approximately for values of k and ℓ which equilibrate the cooperative purchase and grower sales portfolios without considering the values of m and n. Finally, we examine these solutions for their consistency with each other and note the nature of the very small adjustments required to obtain final consistency.

The cooperative and distributor equilibrium solutions were obtained by setting $W_1 = V_1$, $W_2 = V_2$ and $W_3 = V_3$ and solving the equation pairs from Table 6.5 and 6.6 for m and n.[1/] It turned out that the interior offer

---

[1/] $W_3 = 1 - W_1 - W_2$ and $V_3 = 1 - V_1 - V_2$; hence, these values are obtained residually.

curves involving a mix of all three contract types (equation sets 1 in Table 6.5 and 6.6) gave a solution with no negative proportions. The optimal portfolio shares are $W_1 = V_1 = .068$, $W_2 = V_2 = .018$, and $W_3 = V_3 = .914$, with $m = 1.37311$ and $n = .25455$.

Since the offer curves for the cooperative processor sales were approximated with k at 1.35 and ℓ at .38, we began the search for the optimal cooperative-grower exchange proportions by examining the optimal choice for each participant for these parameter values. As indicated in Table 6.2, where $k = 1.35$ and $\ell = .38$ the cooperative would prefer to purchase all tomatoes by sales-minus contract (i.e., $R_3 = .25$). At these values, however, the representative grower prefers a mix of market price and cost-plus sales (see Table 6.1).

To obtain a solution we initially ignored the possible effect of changed values of k and ℓ on the optimal processor portfolio of tomato paste sales contracts. This enabled us to proceed similarly to the processor-distributor equilibrium solution, solving for the values of k and ℓ which result in $R_1 = S_1$, $R_2 = S_2$ and $R_3 = S_3$. The solution is more difficult to obtain in this case, however, because of the very narrow range of contract parameter values which give cooperative purchase solutions involving a mix of all three options. As a result, the data generated by the quadratic programming solutions did not permit us to estimate the interior offer curves for cooperative purchases.1/ However, an examination of the cooperative purchase offer curves and the grower offer curves for the case where the sales-minus option is zero ($R_3 = S_3 = 0$) suggested that

---

1/ As indicated earlier, after the narrowness of the solution range was noted, it would have been possible to have generated solutions for additional contract parameter values within this range. At that point, however, the additional information did not appear to justify the added research cost.

these equations might yield solutions in the positive range. Equating equation (4) in Table 6.4 with (2a) in Table 6.5 yields $k = 1.3367$ and $S_1 = R_1 = .125$ and $S_2 = R_2 = .125$. The border value of ℓ consistent with this solution for the grower is obtained by substituting $k = 1.3367$ and $S_3 = 0$ into equation set (1) in Table 6.4, giving a value of $\ell = .38$. As noted above, we did not estimate an interior offer curve for the cooperative so we cannot verify precisely that $k = 1.3367$ and $\ell = .38$ is consistent with $R_3 = 0$, $R_1 = .125$ and $R_2 = .125$; but inspection of the data in Table 6.2 suggests this likely would hold at least approximately.

Since the initial solutions for m and n were determined for $k = 1.35$, $\ell = .38$ and the separate solutions for equilibrium grower sales and cooperative purchases yielded $k = 1.3367$ and $\ell = .38$, some further reconciliation is needed. Inspection of the data in Table 6.2 suggests that reducing k from 1.35 to 1.3367 would have only a small impact on the optimal choice of V_1, V_2, and V_3. The portfolio solutions obtained above thus may be taken as reasonably close approximation. Had programming solutions been generated for a more extensive set of contract parameter values it would have been possible to increase the precision of the exchange solution. However, we are concerned here more with the general nature of the solutions, rather than specific values, so the added cost of generating the necessary additional observations is not justified by the gain in precision.

The approximate portfolio share solutions obtained above may be converted to a tonnage basis as follows. Since 75,800 tons of paste are sold by the cooperative to the distributor, one may conclude that: 6.9 percent or 5,302 tons of paste are sold under market price contract, 1.0 percent or 758 tons under cost-plus contract, and 92.1 percent or 69,812 tons under sales-minus contract.

Similarly, if one assumes that (a) a total of 4,715 acres of tomatoes (25 percent of total expected acreage requirement of 18,860 acres) are contracted by the cooperative with growers on a nonmember basis, and (b) the portfolio proportions of the grower under investigation are typical (form a weighted average) of the portfolio proportions of all other growers selling to the cooperative on a nonmember basis, then one may conclude that: 12.5 percent or 2,358 acres are contracted to the cooperative on a market price basis, 12.5 percent or 2,358 acres on a cost-plus basis, and no acres on a sales-minus basis. This solution is, of course, applicable only to the particular conditions specified in the model and is applicable only under the assumption of competitive bargaining behavior.

A significant characteristic of the interior solutions is their sensitivity to small variations in values of the contract price parameters and the rather narrow range of parameter space which yields a mixed portfolio. This suggests a basis for some generalizations which will be discussed in the final section of the report.

Thin Market Solutions

It was noted in the first section of the report that a factor contributing to interest in long-term contracts is absence of a reliable process for generating a "market price." This may occur where a market is dominated by one or a very few firms or where there are many private contracts such that a "public" price is not well determined. This is sometimes referred to as a "thin market" problem.

The contract portfolio model provides an approach to analyzing exchange under such thin market situations. We simply delete sales at market price as an alternative in the portfolio. To illustrate, the appropriate offer curve set for the cooperative-distributor exchange solution without market price sales consists of equation (2) in Table 6.5 and (2) in Table 6.6. The offer curves for cost-plus sales contracts are

$$V_2 = 2.7517 + 11.6488m - 73.1395n$$

$$W_2 = -4.4026 - 32.5510m + 193.2960n$$

where $V_1 = W_1 = 0$. Offer curves for sales-minus contracts are determined by $V_3 = 1 - V_2$ and $W_3 = 1 - W_2$.

As in the model involving market price sales or purchase alternatives, a solution is obtained by equating V_2 to W_2 and V_3 to W_3. In this case, however, there is no unique solution. Any values of m and n satisfying the equation obtained from $V_2 - W_2 = 0$, i.e.,

$$7.1543 + 44.1998m - 266.4355n = 0,$$

will yield an equilibrium solution, although the proportions traded by each contract form vary with the parameter values. For example, if $n = .255$ and $m = 1.3753$, $V_2 = W_2 = .121$ and $V_3 = W_3 = .879$. Decreases in n (and correspondingly in m) are associated with increases in the proportions of trade under cost-plus (given this particular set of model specifications). If n is set at .25 and m at 1.3451 the share under cost-plus increases to .136. The value of V_2 and W_2 is reduced to zero if $n \geq .295$.

Note that when the sales at market price alternative is eliminated the solution proportions are much less sensitive to variations in parameter values. The final quantities traded under each nonmarket contract formula and the corresponding parameter values will be determined by relative bargaining strength of the buyer and seller and the reservation floors or ceilings beyond which trade will not occur at all.

VII. SUMMARY AND CONCLUSIONS

The study reported here was initiated as a result of concerns expressed by cooperative executives and others about best procedures and potential benefits associated with long-term product sales contracts. Our objectives were to define the dimensions of the contracting problem, to develop a conceptual framework for analysis, to illustrate the application of the analytical approach within a limited empirical structure, and from this to reach some general conclusions about long-term contracting strategies.

The empirical analysis has focused on the operations of a cooperative fruit and vegetable processor concerned with long-term contract arrangements for the sale of tomato paste to a major reprocessor-distributor. The cooperative also purchases a limited amount of raw tomatoes from nonmember growers and is presumed to be concerned about alternative purchase arrangements for such tomatoes.

Summary of Research Procedures

The first step in the analysis was to identify a set of alternative pricing formulae which have been used or might be considered for use in long-term marketing contracts. Some advantages and disadvantages of each were outlined in Table 1.1. For purposes of the empirical analysis we limited consideration to three alternatives: cost-plus contracts, sales-minus contracts, or market price.

In examining the contracting choices open to the firm, we noted that the contractor need not be limited to a single alternative; rather, he may consider dividing his sales among a set of different types of contracts or sales methods. The problem then may be treated within the framework of portfolio analysis.

The portfolio model expresses expected income (or total profit or net margin) as a function of: shares allocated to each sales or purchase option; total quantity produced and sold; contract price parameter values; and expected values of market price, cost and yield variables. The variance of profit is also expressed as a function of these variables. For given price, cost and yield values, efficient portfolios are defined along "efficiency frontiers" or E-V curves which give the lowest variance for any level of expected profit. It may often turn out that a mix of sales or purchase methods is more efficient than any single option.

The optimal choice among efficient portfolios is determined by the contractor's attitude toward risk, as measured by his utility function for money. A solution may be obtained at the point of tangency of the E-V curve and the contractor's income-risk indifference curves. A more direct procedure is to express expected utility as a function of expected profit and variance of profit. Portfolio solutions which maximize expected utility may be obtained for a range of contract price parameter values. Expressions which relate the portfolio shares to these parameter values are referred to as "offer curves." They provide a means of identifying the regions in the price parameter space where particular contract options are used exclusively, in some mix with other options, or not at all. The offer curves may also be solved simultaneously to determine the conditions of contract equilibrium between buyers and sellers.

Expected values of market price, cost and yield variables were estimated from both secondary data and data supplied by the cooperative, and were projected forward, based on trend estimates, to compute discounted expected value sums over a ten-year contract life. Variances and covariances of these variables were estimated from historical series and subjective estimates of probability distributions from interviews with contracting participants. They were combined to obtain estimates of profit variances.

Quadratic programming was used to estimate E-V frontiers for the representative grower, the cooperative and the distributor for selected values of the contract pricing parameters and other specifications. These frontiers identify efficient contract choices under the conditions specified and provide considerable information and insight as to the trade-off between degree of risk and expected profit. The findings also are independent of any knowledge of attitudes toward risk and so remain valid (given the validity of our data) regardless of our success in measuring and applying individual utility functions.

Since optimal solutions cannot be identified without some knowledge of attitudes toward risk (except for special shapes of E-V curves), we next set out to estimate utility functions for money for a representative set of growers and for representatives of the cooperative. We were unable to obtain the cooperation of the distributor, so a synthetic function was developed for illustrative purposes. Response data for estimating the utility functions were generated through personal interviews using a standard reference contract approach. The interview results suggested variable degrees of risk averse and profit maximizing behavior and provided a basis for empirical estimation.

In fitting smooth functions to the individual utility observations we were constrained by the requirement that the equation form be tractable for further analysis. Two forms were evaluated: quadratic and negative exponential. Both permit optimizing solutions using quadratic programming—the former directly and the latter indirectly. The quadratic function has been criticized for its declining risk aversion with increased money. The exponential function exhibits constant absolute risk, but is not necessarily a better fit to the data. Although such theoretical properties may be of substantial concern for some purposes, we concluded that they were not of major significance in this analysis. What matters most for offer curve shape is the ratio of linear to quadratic components in the function employed to maximize expected utility within the limited range of expected profit levels appropriate to the problem. The positively sloped quadratic functions over the observed range were at least as satisfactory as the exponential functions under these restricted conditions and interpretation and so were used in the remainder of the analysis.

As a base for evaluating utility maximizing solutions (and because of direct interest as well) we extended the expected profit equations so as to define the regions in the contract price parameter space where each sales or purchase option gives maximum expected profit for each participant (grower, cooperative, distributor). These solutions (graphed in Figures 6.1, 6.2, and 6.3) show the best contract solutions for risk-neutral contractors, assuming the other party is agreeable to the terms. If all are risk neutral and have the same perceptions of expected prices and costs, contracting will occur only at contract price parameter values which yield expected profit equivalent to expected market prices.

We next employed the estimated utility functions to determine optimal portfolio solutions for each of the contracting participants for a range of values of contract price parameters. Since the portfolio proportions for each contracting alternative will be zero or one for some values of the contract price parameters, separate solutions must be obtained for each bordered case. For the three sales alternatives considered here, there are seven possible types of solutions: each alternative may be utilized to the maximum permissible level; each may be zero with the other two between zero and the maximum permissible level; or all three may be between zero and the maximum level. The latter is referred to as the interior solution and the other six as boundary solutions.

The solution equations (offer curves) for each of the seven cases turned out to be nonlinear and complex (Appendix B). In the empirical analysis these equations were converted to more manageable forms by developing simplified mathematical approximations to each solution type. This was accomplished by obtaining quadratic programming solutions over a range of contract parameter values and then fitting linear equations to each data set. The fits were generally very close in the sense of having high R^2 values.

Although the empirical estimates of the offer curves are simplified approximations and are specific to the particular utility functions employed, the position on the utility function, and our particular estimates of expected prices and costs and their variances, they provide some general indications of the ways in which contract choices and mixes may vary with changes in contract price parameters. By solving the sets of offer curve approximations for their boundary values (where each sales option reaches its maximum or minimum proportion), we were able to delineate the regions in the contract price parameter space where the sales alternatives are used exclusively, in some mix, or not at all. Comparison of the utility maximizing solutions with the profit maximizing solutions provided an indication of the regions of contract mix and substitution and the limits within which contracting might be expected to occur.

Finally, the offer curves were solved for contract price parameter values and allocations among options to obtain an illustrative equilibrium solution for the specific conditions of this model. A "thin market" solution, in which a clearly defined market price does not exist, was also explored.

General Conclusions

A number of findings are implicit in the research procedures discussed above and explicit in the associated empirical results reported in the body of the report. They need not be repeated here. There are, however, four areas where some further elaboration seems warranted: (1) the impact of degree of risk aversion on optimal contract choices, (2) the role of the open market on problem formulation and solutions, (3) suggestions for self-analysis by potential contractors, and (4) further research needs.

Impact of Degree of Risk Aversion

For the commodity under consideration--tomato paste--the nature of the price and cost variance and the shapes of the associated E-V frontiers are such that for low to moderate levels of risk aversion the optimal portfolio solutions are the profit maximizing solutions. In these cases neither the E-V frontiers nor the risk indifference curves have enough curvature to achieve tangency points within the range of permissible variations in expected income or profit. The optimal solution is at a corner point corresponding to maximum expected profit.

The offer curve under risk neutrality for (say) the seller's cost-plus portfolio share is zero up to some critical value of the cost-plus contract parameter, then jumps to 1.0 (or some other upper limit) at higher values. The buyer's offer curve is reversed and exchange may be expected only at parameter values such that there would be no profit advantage over the expected market price option. Under such conditions long-term contracting is not likely to play a major role as long as purchase or sale at established market prices is a viable alternative.

Offer curve solutions indicate that the sensitivity of portfolios to contract price parameters decreases with the degree of risk aversion. Changes in the magnitude and signs of variances and covariances of the random elements of the profit function also affect the slopes and ranges of the offer curves. The relationship is very complex and difficult to interpret (see Appendix B). It appears, however, that increasing variance or increasing covariance (becoming more positive or less negative) tends to increase the slope coefficients of each offer curve.

With higher levels of risk aversion, cost-plus or sales-minus contracts, or some combination of contracts with market price sales, become more desirable. The ranges of cost-plus markup and sales-minus markdown within which such portfolio mixes are obtained increases with increases in risk aversion. However, in the case studied here, mixed solutions are contained within fairly narrow ranges of parameter values, especially for the sales-minus markdown (see Figures 6.4, 6.5, and 6.6).

Note that mixed solutions do not require all contracting parties to be risk averse. If, for example, the seller is highly risk averse and the buyer risk neutral, a cost-plus contract at less than profit maximizing markup, or some mix of cost-plus and market price, may be preferred by the seller and still be acceptable to the buyer.

Role of the Open Market

Throughout the analysis we have assumed the existence of an open market with sufficient transactions to establish a "market price" for the commodity of concern. Expectations concerning prices in this market play a key role in contracting decisions through their effects on the values of cost-plus markups or sales-minus markdowns which give equal expected profitability of the contract and market price options. If market price expectations increase, higher cost-plus markups and sales-minus markdowns will be required to maintain equal expected profitability. This shift in level may not have much effect on the size of the interior range within which portfolio mix solutions occur unless the shift also is associated with a significant change in the degree of risk aversion.

Our model has also assumed that the buyer and seller have in mind the same value of expected market price. This, of course, need not hold precisely. If, for example, the seller's market price expectation were greater than the buyer's, the seller's profit maximizing break-even markup would return to the buyer a lower expected profitability than market price. This would tend to discourage contracting and would reduce the range of cost-plus markup within which contracting or a contract mix might occur. The reverse would hold if seller market price expectations were below those of the buyer.

For some commodities, open market sales have been increasingly restricted. In such cases, the model developed here may be applied with market price simply deleted as an alternative. We thus have a potential model of exchange in the absence of open market transactions. Explorations of our cooperative-distributor exchange solution with only cost-plus and sales-minus as alternative contract sales options (market price sales excluded) yielded the interesting results that the exchange equilibrium is not unique. That is, there exists a linear combination of cost-plus and sales-minus parameters rather than a single pair, that provide equality of buyer and seller portfolio shares.1/ The range of feasible parameter combinations in this case seems likely to be severely restricted by reservation

---

1/ The combination is linear, given our simplified linear approximations of offer curve solutions.

price considerations, even in the absence of a market price, but it suggests the basis for some pure bargaining. Another characteristic of the thin market (no market price) model is much lower sensitivity of the solution to small variations in contract price parameters. These results suggest that further explorations of thin market situations with more complex models that include additional sales alternatives could be of substantial interest.

Suggestions for Self-Analysis

It is not likely that many firms contemplating long-term sales contracts will have available the analytical resources to explore their alternatives as fully as in this study. There are, however, some procedures suggested by the analysis which may be well within the computational capacity of the firm and which seem essential as a basis for a proper evaluation of the contract choices.

The first step, after identifying the contract and sales alternatives to be considered, is to specify the expected profit equation in a manner as indicated in Tables 3.1, 3.2, and 3.3. With cost and price expectations formed, the profit equation may be used as indicated in Section VI to determine areas within the contract price parameter space which give highest expected returns for each contract alternative (or market price if it is an alternative). If more alternatives than our simple cost-plus and sales-minus options are involved, the parameter space would involve additional dimensions but would be calculated in the same manner.

The next step is to evaluate the attitude of the firm toward risk. If risk aversion is believed to be low, or perhaps even low to moderate, our analysis suggests that a profit maximizing solution still may be appropriate. Negotiations with the other party start from the break-even position with market price and other contract alternatives. If market price is not an alternative, some alteration in profit aspirations may be required if trade is to occur; pure bargaining may become important.

If the firm decision makers seem clearly to be risk averse, some estimates of profit variance will be required. This can be done as indicated in the report. A relatively simple but useful calculation would involve calculation of the profit variance of each alternative if used exclusively. In cases where only two alternatives were considered, it would be easy to calculate expected returns and variances for various allocation proportions. In more complex cases it would be necessary to follow the more involved programming procedures outlined in this report.

An important consideration is the consequence to the firm of the wrong sales choice. For example, the firm might inappropriately make a pure profit maximizing choice when the degree of risk aversion is high. Since utility scales have no cardinal interpretation there is no meaningful way of measuring the utility loss directly. However, the fact that so many of our utility maximizing solutions were near the profit maximizing values and the fact that portfolio mixes were optimal within fairly narrow ranges of contract price parameter variation, suggests that if one is to err it might be best to err toward the profit maximizing solution. This is further supported by the fact that variations in optimal solutions (in the areas where some mix is possible) involve relatively small ranges of variation on the money scale of the utility function.

Further Research Needs

There are several aspects of long-term contractual arrangements, not included in the present analysis, which seem especially worthy of further exploration. This would include extensions to encompass more and different products, more traders, and further consideration of the relation of contract structures to volumes traded. There is need also for further examination of the effects of more complex goal structures such as lexicographic utility, attitudes toward diversification among buyers or sellers, and variable aspiration levels. Attitudes toward risk need to be sampled more widely and alternative means found to measure risk aversion and incorporate it into decision models. Since attitudes toward risk may vary among members of a firm, the resolution of such differences and the impact on contract decisions needs to be examined more fully. While we believe the simplified model presented here has provided some useful information and insights, further exploration seems most desirable if we are fully to understand the impacts of modern developments in exchange systems for agricultural commodities.

APPENDIX A

Described below are the components of grower, cooperative, and distributor E-V and expected utility models. For each modeled firm, mean and variance expressions are first presented. E-V problems are then specified which require substitution of the appropriate mean or variance expression. Finally, expected utility problems are set out which accommodate both quadratic and exponential utility functions. Definitions of terms correspond to Table 2.4.

Grower

Setting $S_4 = .75$ and $S_3 = .25 - S_1 - S_2$, grower profit in Table 2.1 may be expressed as:

(A-1) $\pi_g = -d + A(a - b) + B[c - X_2 + S_1X_1 + S_2kX_2 + (.25 - S_1 - S_2)\ell X_3]$

where

$a = REVF^{a}_{ntom}$

$b = VCF^{a}_{ntom}$

$c = (.75)(z)(NMC)$

$d = FCF_{tom} + FCF_{ntom}$

$A = OAAF$

$B = AAF$

$X_1 = MP^{t}_{tom}Y^{a}_{tom} = MV^{a,sol}_{tom}$

$X_2 = VCF^{a}_{tom}$

$X_3 = REVC.$

Expected grower profit, then, is:

(A-2) $E(\pi_g) = -d + A[\mu_a - \mu_b]$

$$+ B[\mu_c - \mu_2 + S_1\mu_1 + S_2k\mu_2 + (.25 - S_1 - S_2)\ell\mu_3]$$

and variance of grower profit is:

$$
\begin{aligned}
\text{(A-3)}\quad \mathrm{Var}(\pi_g) = {} & [A^2(\sigma_a^2 + \sigma_b^2 - 2\sigma_{ab}) + B^2(\sigma_c^2 + \sigma_2^2 + .0625\ell^2\sigma_3^2 - 2\sigma_{c2} \\
& + .125\ell\sigma_{c3} - .125\ell\sigma_{23}) + 2AB(\sigma_{ac} - \sigma_{a2} - \sigma_{bc} + \sigma_{b2} \\
& + .0625\ell\sigma_{a3} - .0625\ell\sigma_{b3})] \\
& + S_1[2B^2(\sigma_{c1} - \sigma_{12} - \ell\sigma_{c3} + \ell\sigma_{23} + .25\ell\sigma_{13} - .25\ell^2\sigma_3^2) \\
& + 2AB(\sigma_{a1} - \sigma_{b1} - \ell\sigma_{a3} + \ell\sigma_{b3}] \\
& + S_2[2B^2(k\sigma_{c2} - k\sigma_2^2 - \ell\sigma_{c3} + \ell\sigma_{23} + .25k\ell\sigma_{23} - .25\ell^2\sigma_3^2) \\
& + 2AB(k\sigma_{a2} - k\sigma_{b2} - \ell\sigma_{a3} + \ell\sigma_{b3})] \\
& + S_1^2B^2(\sigma_1^2 + \ell^2\sigma_3^2 - 2\ell\sigma_{13}) \\
& + S_2^2B^2(k^2\sigma_2^2 + \ell^2\sigma_3^2 - 2k\ell\sigma_{23}) \\
& + 2S_1S_2B^2(\ell^2\sigma_3^2 + k\sigma_{12} - \ell\sigma_{13} - k\ell\sigma_{23}).
\end{aligned}
$$

E-V curve generation involves calculating, for selected levels $E(\pi_g)°$,

$$
\begin{aligned}
\text{(A-4)}\quad & \min_{S}\ \mathrm{var}(\pi_g) = \alpha_1 + \beta_1'S + S'\Sigma_1 S \\
& \text{s.t. } E(\pi_g) = E(\pi_g)°
\end{aligned}
$$

where α_1 is the first bracketed term in (A-3), β_1 is a column vector of the coefficients of S variables, Σ_1 the matrix of coefficients of S_1^2, S_2^2, and $2S_1S_2$, and S a column vector $[S_1\ S_2]$. Values of the μ_i, σ_i^2 used in grower E-V curve generations are given in Table 3.5, and covariances σ_{ij} in Table 3.6. Grower Frontier #1 sets B = 500 and employs $VCF^{a,o}_{tom}$ moments. Frontier #2 sets B = 500 and utilizes $VCF^{a,\ell}_{tom}$ moments. Frontier #3 assumes B = 1,000 and employs $VCF^{a,o}_{tom}$ moments. All frontiers are solved for A = 800, k = 1.25, ℓ = .385, and assume contracts are signed on an acreage basis ($\widetilde{MV}^a_{pst}$, $\widetilde{MV}^a_{sce}$).

Cooperative

Setting $V_1 = 1 - V_2 - V_3$, and the grower portfolio proportions as above, the cooperative net margin in Table 2.2 may be expressed as:

$$\text{(A-5)} \quad NM_c = e - f - g + D[- X_5 + V_1X_4 + V_2n(X_1 + X_5) + (1 - V_2 - V_3)nX_6 - R_1X_1 - R_2kX_2 - (.25 - R_1 - R_2)\ell X_3]$$

where

$e = REVC_{npst}$

$f = VCC_{npst}$

$g = FCC_{pst} + FCC_{npst}$

$D = AAC$

$X_4 = (1/x)(MP^t_{pst}Y^a_{tom}) = MV^a_{pst}$

$X_5 = (1/x)(NTVCC^t_{pst}Y^a_{tom}) = NTVCC^a_{pst}$

$X_6 = (1/xy)(MP^t_{sce}Y^a_{tom}) = MV^a_{sce}$

V_1, V_2 = paste sales portfolio proportions

R_1, R_2 = tomato purchase portfolio proportions (nonmember), and other variables are defined as previously.

Expected cooperative net margin is, then:

$$\text{(A-6)} \quad E(NM_c) = \mu_e - \mu_f - g + D[-\mu_5 + V_1\mu_4 + V_2m(\mu_1 + \mu_5) + (1 - V_1 - V_2)n\mu_6 - R_1\mu_1 - R_2k\mu_2 - (.25 - R_1 - R_2)\ell\mu_3]$$

and variance of cooperative net margin is:

$$\begin{aligned}\text{(A-7)} \quad Var(NM_c) = {} & [\sigma_e^2 + \sigma_f^2 + D^2(\sigma_5^2 + n^2\sigma_6^2 + .0625\ell^2\sigma_3^2) + 2D(\sigma_{ef} - \sigma_{e5} \\ & + n\sigma_{e6} - .25\ell\sigma_{e3} + \sigma_{f5} - n\sigma_{f6} + .25\ell\sigma_{f3}) + 2D^2(-n\sigma_{56} \\ & + .25\ell\sigma_{35} - .25n\ell\sigma_{36})] \\ & + V_1[2D(\sigma_{e4} - n\sigma_{e6} - \sigma_{f4} + n\sigma_{f6}) + 2D^2(-\sigma_{45} + n\sigma_{56} + n\sigma_{46} \\ & - .25\ell\sigma_{34} - .25n\ell\sigma_{36} - n^2\sigma_6^2]\end{aligned}$$

$$+ V_2[2D(m\sigma_{e1} + m\sigma_{e5} - n\sigma_{e6} - m\sigma_{f1} - m\sigma_{f5} - n\sigma_{f6})$$

$$+ 2D^2(-m\sigma_{15} - m\sigma_5^2 - n\sigma_{56} + mn\sigma_{16} - .25m\ell\sigma_{13}$$

$$+ mn\sigma_{56} - .25m\ell\sigma_{35} - .25n\ell\sigma_{36} - n^2\sigma_6^2)]$$

$$+ R_1[2D(-\sigma_{e1} + \ell\sigma_{e3} + \sigma_{f1} - \ell\sigma_{f3}) + 2D^2(-.25\ell^2\sigma_3^2$$

$$+ \sigma_{15} - \ell\sigma_{35} - n\sigma_{16} - n\ell\sigma_{36} + .25\ell\sigma_{13})]$$

$$+ R_2[2D(-k\sigma_{e2} + \ell\sigma_{e3} + k\sigma_{f2} - \ell\sigma_{f3}) + 2D^2(-.25\ell^2\sigma_3^2$$

$$+ k\sigma_{25} - \ell\sigma_{35} - nk\sigma_{26} - n\ell\sigma_{36} + .25k\ell\sigma_{23})]$$

$$+ V_1^2D^2(\sigma_4^2 + n^2\sigma_6^2 + 2n\sigma_{46})$$

$$+ V_2^2D^2(m^2\sigma_1^2 + m^2\sigma_5^2 + n^2\sigma_6^2 + 2m^2\sigma_{15} - 2mn\sigma_{16} - 2mn\sigma_{56})$$

$$+ R_1^2D^2(\sigma_1^2 + \ell^2\sigma_3^2 - 2\ell\sigma_{13})$$

$$+ R_2^2D^2(k^2\sigma_2^2 + \ell^2\sigma_3^2 - 2k\ell\sigma_{23})$$

$$+ 2V_1V_2D^2(n^2\sigma_6^2 + m\sigma_{14} + m\sigma_{45} - n\sigma_{46} - mn\sigma_{16} - mn\sigma_{56})$$

$$+ 2V_1R_1D^2(-\sigma_{14} + \ell\sigma_{34} + n\sigma_{16} - n\ell\sigma_{36})$$

$$+ 2V_1R_2D^2(-k\sigma_{24} + \ell\sigma_{34} + nk\sigma_{26} - n\ell\sigma_{36})$$

$$+ 2V_2R_1D^2(-m\sigma_1^2 + m\ell\sigma_{13} - m\sigma_{15} + m\ell\sigma_{35} + n\sigma_{16} - n\ell\sigma_{36})$$

$$+ 2V_2R_2D^2(-mk\sigma_{12} + m\ell\sigma_{13} - mk\sigma_{25} + m\ell\sigma_{35} + nk\sigma_{26} - n\ell\sigma_{36})$$

$$+ 2R_1R_2D^2(\ell^2\sigma_3^2 + k\sigma_{12} - \ell\sigma_{13} - k\ell\sigma_{23}).$$

Cooperative E-V curve generation involves a similar constrained variance minimization as in (A-4), where S is replaced by a column vector Z of V's and R's, and the appropriate coefficient substitutions α_2, β_2, Σ_2 are made from (A-7):

(A-8) $\underset{Z}{\text{Min}}\ \text{var}(NM_c) = \alpha_2 + \beta_2'Z + Z'\Sigma_2Z$

$\text{s.t. } E(NM_c) = E(NM_c)°.$

Moments μ_i, σ_i^2, and σ_{ij} used for cooperative E-V curves are for the most part given in Tables 3.5 and 3.6. Instances where there are departures from listed values in Tables 3.5, 3.6, and specifications of all other parameters, are indicated in Table 4.2. As an example, Table 4.2 shows that cooperative Frontier #3 assumes: $m = 1.70$, $n = .22$, $k = 1.25$, $\ell = .08$; no moment alterations from those shown in Tables 3.5; μ_3, σ_3^2 and associated covariances represent total cooperative revenues (as opposed to paste sale revenue only); contracts are on a tonnage basis (MV^a_{sce}, MV^a_{pst} moments used rather than $\tilde{M}V^a_{sce}$, $\tilde{M}V^a_{pst}$ moments); and $R_3 = V_3 = 0$.

Distributor

The distributor/reprocessor profit in Table 2.3 may be represented as:

$$\text{(A-9)}\quad \pi_d = -h + E[X_6 - X_7 - W_1X_4 - W_2m(X_1 + X_5) - (1 - W_1 - W_2)nX_6]$$

where

$h = FCD_{sce}$

$X_7 = (1/xy)(NPVCD^t_{sce}Y^a_{ton}) = NPVCD^a_{sce}$

$E = AAD$

W_1, W_2 = portfolio proportions ($W_3 = 1 - W_1 - W_2$), and other variables are defined as previously.

Expected distributor profit is:

$$\text{(A-10)}\quad E(\pi_d) = -h + E[\mu_6 - \mu_7 - W_1\mu_4 - W_2m(\mu_1 + \mu_5) - (1 - W_1 - W_2)n\mu_6]$$

and variance of distributor profit is:

$$\text{(A-11)}\quad \text{var}(\pi_d) = 2E^2[\sigma_6^2(.5 + .5n^2 - n) + .5\sigma_7^2 + \sigma_{67}(n - 1)]$$

$$+ W_1 2E^2[\sigma_{46}(n - 1) + \sigma_6^2(n - 2n^2) - n\sigma_{67} + \sigma_{47}]$$

$$+ W_2 2E^2[(\sigma_{56}(mn - n) + \sigma_6^2(n - 2n^2) + m\sigma_{17} + m\sigma_{57} - n\sigma_{67}$$
$$+ \sigma_{16}(mn - m)]$$
$$+ W_1^2 2E^2(.5\sigma_4^2 + .5n^2\sigma_6^2 - n^2\sigma_{46})$$
$$+ W_2^2 2E^2(.5m^2\sigma_1^2 + .5m^2\sigma_5^2 + n^2\sigma_6^2 + m^2\sigma_{15} - mn\sigma_{16} - mn\sigma_{56})$$
$$+ 2W_1W_2E^2(2n^2\sigma_6^2 + m\sigma_{14} + m\sigma_{45} - n\sigma_{46} - mn\sigma_{16} - mn\sigma_{56}).$$

Following (A-4) and (A-8), distributor E-V curves are generated by

(A-12) $$\min_{S} \operatorname{var}(\pi_d) = \alpha_3 + \beta_3'W + W'\Sigma_3 W$$
$$\text{s.t. } E(\pi_d) = E(\pi_d)^\circ.$$

Contract parameter values m, n used in each distributor frontier are listed in Table 4.4. Note that in Distributor Frontier #3, σ_4^2 and all covariances with a 4 subscript are set at zero. In the same frontier the value μ_4 is also set at 15 percent less than that listed in Table 3.5. All distributor solutions assume contracts are signed on a tonnage basis; that is moments employed correspond to MV_{pst}^a, MV_{sce}^a in Tables 3.5 and 3.6.

Expected Utility Problems

Expected utility maximizations for grower and distributor, and some of those for the cooperative, are solved under the assumption that decision makers have quadratic utility functions. In general form suited to any of the three firms, the problem is stated as:

(A-13) $$\text{Max } E[U(\pi)] = b\mu_\pi - c(\mu_\pi^2 + \sigma_\pi^2)$$

where

$$U = a - b\pi - c\pi^2, \; b, c > 0.$$

Profit moments μ_π and σ_π^2 are substituted from (A-2), (A-3), (A-6), (A-7), (A-10), (A-11) as appropriate. The expression μ_π^2 is developed by replacing each σ_i^2 in (A-3), (A-7), or (A-11) with μ_i^2, and each σ_{ij} in these equations with $\mu_i\mu_j$.

Expected utility maximization under exponential utility has the simpler form

(A-14) $\text{Max } E[U(\pi)] = \mu_\pi - \lambda\sigma_\pi^2, \quad \lambda > 0.$

where

$$U = K - \theta \exp[-\lambda M], \theta, \lambda > 0.$$

Owing to the absence of μ_π^2, optimal programs in (A-14) are much less sensitive to expected acreage requirements B, D, and E than are optimal programs in (A-13).

Programs (A-13) and (A-14) are not specified here with a constraint since portfolio proportions S_3, V_3, R_3, and W_3 are represented above as residuals of the remaining proportions. Actual program routines included these four variables, and utilized corresponding linear constraints $S_1 + S_2 + S_3 = .25$; $V_1 + V_2 + V_3 = 1$; $R_1 + R_2 + R_3 = .25$; $W_1 + W_2 + W_3 = 1$.

APPENDIX B

Optimizing Solutions for the Expected Utility Functions

A. Grower solution

1. Expected utility function - quadratic utility

$$E[U(\pi_g)] = b\ E(\pi_g) - C[E(\pi_g)]^2 - C\ \mathrm{var}(\pi_g)$$

$$E(\pi_g) = a_0 + a_1S_1 + a_2S_2$$

$$\begin{aligned}\mathrm{var}(\pi_g) = {} & (b_{00} + b_{01}\ell + b_{02}\ell^2) + (b_{10} + b_{11}\ell + b_{12}\ell^2)S_1 \\ & + (b_{21}k + b_{22}\ell + b_{23}\ell^2 + b_{24}k\ell)S_2 + (b_{30} + b_{31}\ell + b_{32}\ell^2)S_1^2 \\ & + (b_{41}k^2 + b_{42}\ell^2 + b_{43}k\ell)S_2^2 \\ & + (b_{51}\ell^2 + b_{52}k + b_{53}\ell + b_{54}k\ell)S_1S_2\end{aligned}$$

where (using Appendix A notation)

$$a_0 = -d + A[\mu_a - \mu_b] + B\mu_c - B\mu_2 + .25B\ell\mu_3$$

$$a_1 = (\mu_1 - \ell\mu_3)B$$

$$a_2 = (k\mu_2 - \ell\mu_3)B$$

$$\begin{aligned}b_{00} = {} & A^2(\sigma_a^2 + \sigma_b^2 - 2\sigma_{ab}) + B^2(\sigma_c^2 + \sigma_2^2 - 2\sigma_{c2}) \\ & + 2AB(\sigma_{ac} - \sigma_{a2} - \sigma_{bc} + \sigma_{b2})\end{aligned}$$

$$b_{01} = .125(\sigma_{c3} - \sigma_{23}) + .0625(\sigma_{a3} - \sigma_{b3})2AB$$

$$b_{02} = .0625\sigma_3^2B^2$$

$$b_{10} = 2B^2(\sigma_{c1} - \sigma_{12}) + 2AB(\sigma_{a1} - \sigma_{b1})$$

$$b_{11} = [.25\sigma_{13} - \sigma_{c3} + \sigma_{23}]2B^2 + 2AB(\sigma_{b3} - \sigma_{a3})$$

$$b_{12} = -.25\sigma_3^2(2B^2)$$

$$b_{21} = 2B^2[\sigma_{c2} - \sigma_2^2] + 2AB(\sigma_{a2} - \sigma_{b2})$$

$$b_{22} = 2B^2[\sigma_{23} - \sigma_{c3}] - 2AB(\sigma_{a3} - \sigma_{b3})$$

$$b_{23} = -.25\sigma_3^2 2B^2$$

$$b_{24} = .25\sigma_{23} 2B^2$$

$$b_{30} = B^2\sigma_1^2$$

$$b_{31} = -2B^2\sigma_{13}$$

$$b_{32} = \sigma_3^2 B^2$$

$$b_{41} = B^2\sigma^2$$

$$b_{42} = B^2\sigma_3^2$$

$$b_{43} = -2B^2\sigma_{23}$$

$$b_{51} = 2B^2\sigma_3^2$$

$$b_{52} = 2B^2\sigma_{12}$$

$$b_{53} = -2B^2\sigma_{13}$$

$$b_{54} = -2B^2\sigma_{23}.$$

2. First order conditions for maximum

$$\begin{aligned}\frac{\partial E[U(\pi_g)]}{\partial S_1} &= ba_1 - 2C(a_0 + a_1S_1 + a_2S_2)a_1 - C(b_{10} + b_{11}\ell + b_{12}\ell^2)\\ &\quad - 2C(b_{30} + b_{31}\ell + b_{32}\ell^2)S_1\\ &\quad - C(b_{51}\ell^2 + b_{52}k + b_{53}\ell + b_{54}k\ell)S_2 = 0\end{aligned}$$

$$\begin{aligned}\frac{\partial E[U(\pi_g)]}{\partial S_2} &= ba_2 - 2C(a_0 + a_1S_1 + a_2S_2)a_2\\ &\quad - C(b_{21}k + b_{22}\ell + b_{23}\ell^2 + b_{24}k\ell)\\ &\quad - 2C(b_{41}k^2 + b_{42}\ell^2 + b_{43}k\ell)S_2\\ &\quad - C(b_{51}\ell^2 + b_{52}k + b_{53}\ell + b_{54}k\ell)S_1 = 0.\end{aligned}$$

Simplifying,

$$\frac{\partial E[U(\pi_g)]}{\partial S_1} = A_{10} + A_{11}S_1 + A_{12}S_2 = 0$$

$$\frac{\partial E[U(\pi_g)]}{\partial S_2} = A_{20} + A_{21}S_1 + A_{22}S_2 = 0$$

where

$$A_{10} = (ba_1 - 2Ca_0a_1 - Cb_{10}) - Cb_{11}\ell - Cb_{12}\ell^2$$

$$A_{11} = - 2C(a_1^2 + b_{30}) - 2Cb_{31}\ell - 2Cb_{32}\ell^2$$

$$A_{12} = - 2Ca_1a_2 - C(b_{52}k + b_{53}\ell + b_{51}\ell^2 + b_{54}k\ell)$$

$$A_{20} = ba_2 - 2Ca_0a_2 - C(b_{21}k + b_{22}\ell + b_{23}\ell^2 + b_{24}k\ell)$$

$$A_{21} = A_{12}$$

$$A_{22} = - 2Ca_2^2 - 2C(b_{41}k^2 + b_{42}\ell^2 + b_{43}k\ell)$$

3. Solutions $[0 \leq S_1, S_2, .25-S_1-S_2 \leq .25]$

(1) Interior solution $[0 < S_1, S_2, .25-S_1-S_2 < .25]$

$$S_1^0 = \frac{A_{12}A_{20} - A_{10}A_{22}}{A_{11}A_{22} - A_{12}^2}$$

$$S_2^0 = \frac{A_{10}A_{21} - A_{11}A_{20}}{A_{11}A_{22} - A_{12}^2}$$

(2) Boundary solutions

(a) $S_1 = 0, \quad 0 < S_2, S_3 < .25$

$$S_2^0 = - \frac{A_{20}}{A_{22}}, \quad S_3^0 = .25 + \frac{A_{20}}{A_{22}}$$

(b) $S_2 = 0, \quad 0 < S_1, S_3 < .25$

$$S_1^0 = - \frac{A_{10}}{A_{11}}, \quad S_3^0 = .25 + \frac{A_{10}}{A_{11}}$$

(c) $S_3 = 0, \quad 0 < S_1, S_2 < .25$

(all terms with ℓ and μ_3 drop)

$$S_1^0 = \frac{A_{12}^1A_{20}^1 - A_{10}^1A_{22}^1}{A_{11}^1A_{22}^1 - (A_{12}^1)^2}$$

where $A^1_{10} = bB\mu_1 - 2C[-d + A(\mu_a - \mu_b) + B(\mu_c - \mu_2)]B\mu_1$
$- Cb_{10}$

$$A^1_{11} = -2C(\mu_1^2 + b_{30})$$

$$A^1_{12} = -2C\mu_1\mu_2kB^2 - Cb_{52}k$$

$$A^1_{20} = bBk\mu_2 - 2C[-d + A(\mu_a - \mu_b) + B(\mu_c - \mu_2)]Bk\mu_2 - Cb_{21}k$$

$$A^1_{22} = -2Ck^2(\mu_2^2B^2 + b_{41})$$

(d) $S_1 = .25, \quad S_2, S_3 = 0$

$$-\frac{A_{10}}{A_{11}} \geq .25, \quad -\frac{A_{20}}{A_{21}} \geq .25$$

(e) $S_2 = .25, \quad S_1, S_3 = 0$

$$-\frac{A_{10}}{A_{12}} \geq .25, \quad -\frac{A_{20}}{A_{22}} \geq .25$$

(f) $S_3 = .25, \quad S_2, S_3 = 0$

$$A_{10} = 0, \quad A_{20} = 0$$

B. Distributor and processor solutions

The solution for the distributor is identical to the grower model except that S_i is replaced by W_i and the numerical values (.25, .125, .0625) are replaced by 1.0.

The processor model is similar in format but is complicated by the inclusion of both V_i and R_i variables. This adds greatly to the number of covariance and cross product terms and the number of boundary solutions required. In the empirical analysis it was found that solutions for V_i and R_i could be obtained independently without significantly affecting the outcomes. The problem then reduces to two optimizing models (for purchases and sales) similar in format to the grower case.

APPENDIX C

Procedures for Constructing Historical Series of Cost Variables

Tomato Farm Inputs (VCF^{a}_{tom})

Labor

Annual labor hours per acre of tomatoes, broken down by farm operation and job classification, are reported for each of the six principal counties in Table C1. These labor figures are taken directly from University of California Cooperative Extension reports dated 1972-1974.

The Extension Service studies divide costs among labor, fuel and repairs, and materials. The labor component in the repairs part of the fuel and repairs category was estimated as shown in Table C2. First, fuel requirements were taken from Table C3, column (6), and fuel costs per acre for each county were calculated as shown in column (3), Table C2. Fuel costs were deducted from fuel and repairs figures to leave repair costs only (columns 2,3). Repair costs (column 4) were then divided by the current farm wage rates for mechanics ($3.50 in 1973, $3.00 in 1970) to give an inputed labor time for repairs. The results (column 6) are reported under the "all repairs" farm operation for each county, Table C1. This procedure introduces some inaccuracy since repair costs are only partly due to labor, the balance to parts. (From grower interviews, the allocation is approximately 50 percent each.) However, some method had to be found to incorporate parts costs into the total cash cost series, and such a series in its own right is awkward to construct. Besides, it is probable that labor wages have an important influence on parts and installation costs.

TABLE C1

Annual Labor Hours Per Acre to Cultivate and Harvest Processing Tomatoes in California, Selected Counties, by Operation and Wage Class[a]

Wage class / Operation		Mechanic	Harvester operator sorter supervisor	Tractor operator	Irrigator	Unskilled	Totals
Seedbed	(SJ)			3.85			3.85
	(Y)			3.40			3.40
	(SOL)			3.85			3.85
	(SB)			2.74			2.74
Plant	(SJ)			1.10			1.10
	(Y)			.88			.88
	(SOL)			.84			.84
	(SB)			.42			.42
Grow	(SJ)			2.76[b]	10.62	15.25	28.63
	(Y)			3.13	10.50	9.50	23.13
	(FR)			9.00[c]	10.85	15.00	25.85
	(SOL)			2.64[b]	10.50	16.50	29.64
	(SB)			2.06[b]	14.00	12.00	28.06
Harvest, bins[d]		3.30	5.9	10.25 (SJ)		40.50 (SJ)	59.95 (SJ)
				8.80 (Y)		40.50 (Y)	58.50 (Y)
				11.77 (FR)		46.56 (FR)	67.53 (FR)
operator			3.2	10.25 (SOL)		45.90 (SOL)	65.35 (SOL)
supervisor			2.7	8.90 (SB)		39.90 (SB)	57.80 (SB)
mechanic		3.30					
All repairs[e]	(SJ)	16.91					16.91
	(Y)	11.18					11.18
	(FR)	13.22					13.22
	(SOL)	14.38					14.38
(imputed)	(SB)	10.43					10.43
Totals	(SJ)	20.21	5.9	17.96	10.62	55.75	110.44
	(Y)	14.48	5.9	16.21	10.50	50.00	97.09
	(FR)	16.52	5.9	20.77	10.85	61.56	115.60
	(SOL)	17.68	5.9	17.58	10.50	62.40	114.06
	(SB)	13.78	5.7	14.12	14.00	51.90	99.45

[a] County observations are: (SJ) = San Joaquin, 1973; (Y) = Yolo, 1970; (FR) = Fresno, 1972; (SOL) = Solano, 1970; (SB) = San Benito, 1973.

[b] Includes 8 hours imputed to fertilizer application (from Yolo sheet).

[c] This figure includes all cultural operations for Fresno County.

[d] Mechanic and harvester operator rates are nearly identical for all counties.

[e] Includes cost of parts divided by repair labor wage per hour.

Source: University of California Agricultural Extension Service.

TABLE C2

Fuel and Repair Costs Associated with Producing Processing Tomatoes in California, 1970-1973, by Selected County, Including Procedure to Compute Repair Labor Hours Per Acre

	(1)	(2)	(3)	(4)	(5)	(6)
County	Fuel and repairs cost	Less fuel	Fuel computation	Total repair bill	Imputed hourly labor cost, repairs	Imputed labor hours per acre, repairs
	dollars per acre			dollars per acre	dollars per hour	hours per acre
San Joaquin, Contra Costa, Stanislaus (1973)	79.21	20.00	(125 gal) (16¢/gal)	59.21	3.50	16.91
Yolo (1970)	51.47	17.92	(112 gal) (16¢/gal)	33.55	3.00	11.18
Fresno (1972)	NA	21.44	(134 gal) (16¢/gal)	NA	NA	13.22
Solano (1970)	62.97	19.84	(124 gal) (16¢/gal)	43.13	3.00	14.38
San Benito (1973)	54.42	17.92	(112 gal) (16¢/gal)	36.50	3.50	10.43

Sources: Columns (1), (5): University of California Agricultural Extension Service.
Column (3): Price per gallon from Reed, gallons from Table C5.

TABLE C3

Input Units Required Annually to Produce One Acre of Processing Tomatoes in California, by Selected County, 1970-1973

Input / County	(1) Labor[b]	(2) Seed	(3) Fertilizer	(4) Insecticide	(5) Herbicide	(6) Diesel	(7) Water	(8) Proportion of cooperative tomato acreage in each county
	hours	pounds	pounds, gallons	gallons	pounds	gallons	acre-feet	
San Joaquin, Stanislaus, Contra Costa (1973)	110.44	.75	100 lbs N[a]	NA	6.4	125.0	5.00	34.68
Yolo (1970)	97.09	1.00	75 lbs N[a] 10 gals 8-24-0	1.25 gals Lanate .1875 gals Dieldrin	4.00	112.0	5.00	19.86
Fresno (1972)	115.60	1.00	90 lbs N[a] 50 lbs P_2O_5	NA	3.0	134.0	3.00	17.14
Solano (1970)	114.06	1.00	125 lbs N[a]	NA	2.0	124.0	5.00	16.46
San Benito (1973)	99.45	.33	400 lbs 6-20-20 400 lbs 21-0-0	NA	2.0	112.0	2.33	11.86
Weighted average	107.966	.834	--	--	3.697	122.25	4.34	--

[a] Actual element. Usually applied as ammonium hydroxide.

[b] Includes cost of parts divided by repair labor wage per hour.

Source: University of California Agricultural Extension Service.

Total per-acre labor hours by county are summarized on Table C3, column (1). A weighted average of these is taken, where the weights represent the proportionate acreage share of each county in cooperative membership (column 8). This weighted average (107.97) is then employed as labor input coefficient for the final total variable cost series.

The labor wage time series, 1951-1974, is constructed by forming a weighted average tomato farm labor wage for 1973 and multiplying this by an index (1973=100) of unskilled tomato farm labor rates, 1951-1974. The result is an estimate of the weighted average hourly wage paid to tomato farm employees in each of these years, precisely what is needed in conjunction with a per-acre labor hours coefficient that combines labor of all job classifications.

Calculation of the weighted average hourly wage is reproduced on Table C4. Average imputed hourly wages paid in San Joaquin County for each job class (column 3) are multiplied by the corresponding average proportions of per-acre labor time allocated to each job class (column 5). These multiples are summed (column 6) to yield the weighted average hourly cash wage in 1973 ($2.86). This cash wage is multiplied by 1.12 to account for social security, workmen's compensation, and bonuses. The total is $3.21 per hour.

The price index series used to move this total backward in time is derived primarily from unskilled wage data reported by the State of California, Department of Employment (Human Resources Development). Mid-August quotes from this source for each county were compiled and an index (1967=100) derived from the simple county averages. Since this series begins only in 1960, it was extended to 1951 from data on California farm wage rates reported by the State of California, Agricultural Labor Commission, Exhibit E, p. 17. The corresponding wage series (index times 1973 average wage) is presented on Table C6, column (1).

TABLE C4

Calculation Procedure Used to Obtain Weighted Average Hourly Wage Paid to Workers on Processing Tomato Farms, Lower Sacramento and Upper San Joaquin Valleys, California, 1973

	(1)	(2)	(3)	(4)	(5)	(6)
Wage class	Wage scale, San Joaquin, 1973	Ratio of each wage rate to unskilled rate	Presumed wage rates if unskilled rate is $2.46	Average hours devoted to work in these classes	Proportion of total work hours occupied by each class	Column (3) times column (5)
	dollars per hour		dollars per hour	hours		
Supervisor and mechanic	3.50	1.6667	4.1001	16.524	.154	.6314
Harvester operator, sorter supervisor	3.00	1.4286	3.5144	5.86	.0546	.1919
Tractor and forklift drivers	2.50	1.1905	2.9286	17.328	.1614	.4727
Irrigators	2.25	1.0714	2.6356	11.294	.1052	.2773
Unskilled	2.10	1.0000	2.46	56.322	.5248	1.291
				Σ=107.328	Σ=1.0000	Σ=2.8643[a]

---

[a] This is the weighted average hourly wage rate, 1973. Adding 12 percent for social security, workmen's compensation, etc., we obtain $3.21.

Sources by column:

(1)-(5): California Agricultural Extension Service
(6): Column (3) times column (5).

TABLE C5

Calculation Procedure Used to Obtain Gallons of Diesel Required Annually in Producing an Acre of Processing Tomatoes, California, 1970-1973, Selected Counties

County	(1) Average horsepower used on farm	(2) Average gallons diesel per hour[a/] per horsepower	(3) Average gallons diesel per hour	(4) Farm vehicle operating hours per acre	(5) Gallons of diesel used per acre
San Joaquin (1973)	80	.070	5.6	22.34	125
Yolo (1970)	80	.070	5.6	20.17	112
Fresno (1972)	80	.070	5.6	24.00	134
Solano (1970)	80	.070	5.6	22.09	124
San Benito (1973)	80	.070	5.6	20.00	112

[a/] At 75 percent of maximum horsepower.

Sources by column:

(1), (2): Reed.
(3): Column (1) times column (2).
(4): University of California Agricultural Extension Service.
(5): Column (3) times column (4).

TABLE C6

Per-Acre Input Coefficients and Annual Unit Input Prices, Processing Tomato Farms, California, 1951-1974

Year \ Input and units per acre, 1973	(1) Labor (107.97 hours)	(2) Seed (1 lb)	(3) Fertilizer (50 gals N, 10 gals 8-24-0)	(4) Insecticide (1.25 gals Lanate .137 gals Dieldrin)	(5) Herbicide (4 lbs) Trefmid)	(6) Diesel (122.25 gals)	(7) Water (4.34 acre-feet)
	dollars per hour	dollars per pound	dollars per gallon	dollars per gallon	dollars per pound	dollars per gallon	dollars per acre-foot
1951	1.0986	14.86	.1819	40.38	5.3285	.14520	3.306
1952	1.1621	19.10	.1853	35.84	5.9097	.14623	3.333
1953	1.1892	16.13	.1884	21.45	5.0000	.15118	3.340
1954	1.1795	14.48	.1860	23.55	4.3500	.15327	3.431
1955	1.2065	12.90	.1843	23.55	4.0206	.15716	3.269
1956	1.2571	13.97	.1843	21.98	3.6330	.16432	3.175
1957	1.2744	13.79	.1865	19.71	3.7299	.17343	3.218
1958	1.2940	13.85	.1877	19.01	3.8752	.15755	3.240
1959	1.3305	14.36	.1857	20.06	3.8752	.16133	3.253
1960	1.3824	13.79	.1883	20.06	3.8552	.15204	3.289
1961	1.4475	15.12	.1915	18.40	3.8752	.15945	3.305
1962	1.3171	19.03	.1938	18.23	3.7009	.15730	3.318
1963	1.3954	17.46	.1934	16.22	3.3426	.15780	3.292
1964	1.4344	16.07	.1939	14.48	3.3426	.14529	3.263
1965	1.8126	16.00	.1962	14.82	3.2844	.15434	3.253
1966	1.9040	15.00	.1971	15.70	3.1486	.15747	3.237
1967	1.8648	15.00	.1986	15.61	3.2165	.16800	3.250
1968	2.1387	15.00	.1936	15.20	3.1322	.17112	3.279
1969	2.3213	12.00	.1822	14.82	3.0541	.17195	3.315
1970	2.3342	10.00	.1880	16.79	3.4600	.17853	3.405
1971	2.4647	11.00	.2026	17.45	3.5960	.18480	3.692
1972	2.4517	12.00	.2026	17.47	3.5993	.18698	3.949
1973	3.2080	12.00	.2262	17.59	3.6253	.23470	4.202
1974	3.2472	13.00	.3053	18.35	3.7816	.47107	5.506

Sources by column: In most cases, price series were developed first as index series then converted to actual prices through use of a base price. Following are sources of index series and base prices. 1974 values are early estimates.

	Index series	Base price
(1)	1951-1959: Calif. Ag. Lab. Com.; 1960-1974: Calif.: *Weekly Farm Labor Rep.*	California Agricultural Labor Commission, p. 17
(2)	1951-1964: Proportionate to one-year lagged tomato price changes.	Garner Seed Co., Woodland, CA (supplied prices 1965-1974).
(3)	U.S. Department of Labor: Wholesale Price Indexes (WPI)	Mel Zobel, Agr. Extension Service, Woodland.
(4)	1951-1966: USDA: *Ag. Stats.*, p. 585; 1967-1974: *WPI*, "Pesticides".	Mel Zobel, Agr. Extension Service, Woodland.
(5)	1951-1966: USDA: *Ag. Stats.*, p. 585; 1967-1974: *WPI*, "Pesticides".	Mel Zobel, Agr. Extension Service, Woodland.
(6)	*WPI*, "Middle Distillate".	U.S.D.A.: *Agricultural Prices*
(7)	Tri/Valley Growers, San Francisco, California	Mel Zobel, Agr. Extension Service, Woodland.

Seed, Pesticides, and Fertilizer

Seed, insecticide, herbicide, and fertilizer uses per acre are summarized on Table C3, columns (2)-(5). Average seed use is .84 pounds per acre but the assumption of 1 pound per acre was considered more typical (modal). Only Yolo County reported insecticide quantities (1.25 gallons Lanate, .1875 gallons Dieldrin). For herbicide, the Yolo County use of 4 pounds Trefmid per acre, near the six-county average, was employed. The Yolo County figures for fertilizer (50 gallons or 75 pounds nitrogen; 10 gallons 8-24-0) were also employed since they are 1975 specifications and fertilizer use is so varied across counties and farms.

The price series for seed, 1965-1974, was provided by a seed distributor (Table C6, column 2). The 1951-1964 part of this series was constructed on the assumption, suggested by several distributors, that seed prices change proportionately with one-year-lagged tomato prices. The wholesale price index for pesticides, U.S. Department of Labor, Wholesale Price Indexes, was used for both insecticide and herbicide price indexes in the period 1967 to 1974. From 1951 to 1966, the insecticide index follows price movements of 2,4-D, and the herbicide series, price movements of DDT [U.S.D.A., Agricultural Statistics]. Current prices of the indicated insecticides and herbicide were then used to convert index series to price series (Table C6, columns 4, 5). The price index for fertilizer was also taken from Bureau of Labor Statistics annual wholesale price summaries ("mixed fertilizer") and converted to a price series (Table C6, column 3) by applying the current weighted average price of nitrogen (ammonium hydroxide) and 8-24-0. The weights for nitrogen and 8-24-0 are .833, .167, respectively.

Diesel and Water

Per-acre diesel fuel use by county, given on Table C3, column (6), are calculated as shown on Table C5. Average gallons of diesel use per hour [Reed] are multiplied by farm vehicle operating hours per acre, as reported in the county cost studies, to yield an estimate of gallons of diesel consumed per acre. A six-county weighted average of these coefficients (122.25 gallons per acre) is then formed as indicated.

Water use figures in feet per acre are obtained directly from Agricultural Extension Service cost sheets. These are shown on Table C3, column (7), together with their six-county weighted average (4.34).

The diesel fuel price series, as shown on Table C6, column (6) was constructed by applying the 1974 average California farmer price for diesel to the Bureau of Labor Statistics price index series for middle distillate.

The acre-foot cost of irrigation water is assumed to vary directly with the agricultural rate for electricity (pumping cost). To the BLS price index series for electricity was therefore applied the 1974 acre-foot cost of irrigation ($5.93 at 1,500 gallons per minute) to obtain the time series of irrigation cost per acre foot (Table C6, column 7). The BLS series for industrial electricity was too short to be useful.

Land Rent

Land Rent is a cash cost for share-lessees only. Usually rent is calculated as a percentage of tomato revenue, although in some cases it is a flat charge. For owner-operators, rent is the opportunity cost of land, or the market value of land times the annual rate of return the owner thinks he could earn with its market value by investment in an alternative, risk-comparable enterprise. This is a noncash cost and normally classified with fixed charges, but it is included here as a cash cost for purposes of comparison.

Construction of rent series for owner-operator and share-lessee are shown in Table C7. For owner-operators, a series of estimated irrigated land values in Yolo County (column 2) is computed from a published index series (column 1) and multiplied by three-month Treasury Bill[1] rates to give the imputed annual opportunity cost of irrigated land (column 4). For share-lessees, annual average tomato revenues in Solano County (our best estimate of farmer revenue variability), columns (5) and (6), are multiplied by estimated average lessor shares (column 7) to give an estimated average annual cash rent (column 8).

Total Costs

Total cash cost series and fixed costs for owner-operator and share-lessee tomato growers are summarized on Table C8. Principal costs, the same for both growers, are found by multiplying input coefficients with input price series in Table C6 and summing across rows. Miscellaneous costs include office expenses, road maintenance, and taxes; the miscellaneous cost series are formed by multiplying a constant factor times principal costs.[2] Imputed and cash rents are taken from Table C7. Total cash costs then comprise principal, miscellaneous, and rent costs. The fixed costs listed apply to 1974 only and are taken from the Cooperative Extension cost study for San Joaquin County. No series was required for fixed costs since they are considered nonstochastic in the decision models.

Paste Processing Inputs ($NTVCC^{tr}_{pst}$)[3]

---

[1] Growers frequently purchase treasury bills as a side investment, and treasury bill interest rates are a good proxy for low risk, short term investment return.

[2] The factors are .2119 for owner-operators and .1753 for share-lessees. These are the average rates of miscellaneous to principal costs in county tomato cost studies. The rates differ since share-lessees do not pay land tax.

[3] Input cost series are expressed in index form to honor the source's request for privacy.

TABLE C7

Calculation Procedure Used to Obtain Per-Acre Land Rent Estimates for Processing Tomato Farms, California, 1951-1974, Owner-Operators and Share-Lessees

Year	(1) Index of values of irrigated land	(2) Imputed value of irrigated land, Yolo County	(3) Interest rates on 3-month Treasury Bills	(4) Imputed rent	(5) Market price of processing tomatoes F.O.B. farm	(6) Solano County yields	(7) Estimated average share of lessor	(8) Estimated cash rent, Solano County
	1967=100	dollars per acre	percent per annum	dollars per acre	dollars per ton	tons per acre		dollars per acre
1951	47	640.91	1.552	9.95	30.20	18.68	.12	67.70
1952	54	736.37	1.766	13.00	25.50	20.95	.12	64.11
1953	55	750.00	1.931	14.48	22.90	18.68	.12	51.33
1954	54	640.91	.953	6.11	20.40	19.51	.12	47.76
1955	57	777.27	1.753	13.63	22.80	20.20	.13	59.87
1956	62	845.46	2.658	22.47	22.70	20.60	.13	60.79
1957	67	913.64	3.267	29.85	21.90	16.12	.13	45.89
1958	72	981.82	1.839	18.06	22.70	18.21	.13	53.74
1959	79	1,077.28	3.405	36.68	21.80	14.92	.14	45.54
1960	84	1,145.46	2.928	33.54	23.40	17.52	.14	57.40
1961	89	1,213.64	2.378	28.86	30.10	14.15	.14	59.59
1962	92	1,254.55	2.778	34.85	27.60	18.76	.14	72.49
1963	95	1,295.46	3.157	40.90	25.40	19.26	.15	73.38
1964	97	1,322.73	3.549	46.94	25.30	24.80	.15	94.12
1965	99	1,350.00	3.954	53.38	35.40	20.34	.15	108.01
1966	100	1,363.64	4.881	66.56	30.00	20.43	.15	91.93
1967	100	1,363.64	4.321	58.92	38.70	17.77	.16	110.03
1968	104	1,418.19	5.339	75.72	35.20	22.43	.16	126.33
1969	100	1,363.64	6.677	91.05	27.20	22.35	.16	97.27
1970	100	1,363.64	6.458	88.06	25.20	22.60	.16	91.12
1971	99	1,350.00	4.348	58.70	28.00	24.41	.17	116.19
1972	100	1,363.64	4.071	55.51	28.00	23.86	.17	113.57
1973	98	1,336.37	7.041	94.09	35.00	23.57	.17	140.24
1974	110	1,500.00	7.246	108.69	56.80	20.50	.17	197.95

Sources by column:

(1) "Farm Real Estate Market Developments," ERS, USDA.
(2) Derived from assumption that 1974 value is $1,500 (Yolo County Assessor).
(3) "Economic Indicators," Council of Economic Advisors.
(4) Column (2) times column (3).
(5) "Vegetables-Processing," SRS, USDA.
(6) _Ibid_.
(7) Estimates by Mel Zobel, Farm Advisor, Yolo County.
(8) Column (5) times column (6) times column (7).

Labor

Annual cannery wages paid to production line workers were supplied by the cooperative for the years 1964-1974. These were converted to weighted average wages paid to all cannery workers by multiplying each with a constant index. This index is the ratio of the weighted average hourly wage to the production worker wage in 1975.

To move the series back to the 1951 starting point employed in this study, an index of California wages paid in fruit and vegetable processing, reported in U.S. Department of Labor, *Employment and Earnings*, p. 43, SIC 2032-5,7, was recorded for the period 1951-1964. The 1964 actual wage was then extended backward proportionately with this index series. A final series, expressed in index form, 1974=100, is reported on Table C9, column (1). This wage index increases substantially and monotonically over the 24-year sample period. The 1974 cannery wages are nearly 320 percent higher than the 1951 wages.

Drum

A monthly and annual index of prices for 55-gallon barrels is reported in the *Wholesale Price Index* series, U.S. Department of Labor. But the 55-gallon barrel classification was begun only recently, and a larger class index, "barrels, drums, and pails," was used instead. During 1951-1953, the "metal containers" class was used. The index series, given on Table C9, column (2), was converted to an actual price series by multiplying it with recent barrel prices.

Electricity and Gas

Wholesale price index series were also employed in conjunction with current actual prices to construct 1951-1974 actual price series for

TABLE C8

Summary of Annual Per-Acre Costs to Produce Processing Tomatoes, Weighted Average of Selected Six-County Area, California, 1951-1974, Owner-Operator and Share-Lessee, Assuming 1973 Production Technology

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	Owner operator					Share lessee				
Year	Principal cash costs	Miscellaneous cash costs	Rent imputation	Total cash costs	Fixed costs	Principal cash costs	Miscellaneous cash costs	Rent	Total cash costs	Fixed costs
	dollars					dollars				
1951	255.84	54.21	9.95	320.00		255.84	44.85	67.70	368.39	
1952	263.20	55.77	13.00	331.97		263.20	46.14	64.11	373.45	
1953	239.63	50.78	14.48	304.89		239.63	42.01	51.33	332.97	
1954	237.87	50.40	6.11	294.38		237.87	41.70	47.76	327.33	
1955	237.55	50.34	13.63	301.52		237.55	41.64	59.87	339.06	
1956	240.75	51.01	22.47	314.23		240.75	42.20	60.79	343.74	
1957	240.99	51.07	29.85	321.91		240.99	42.25	45.89	329.12	
1958	240.97	51.06	18.06	310.09		240.97	42.24	53.74	336.95	
1959	247.33	52.41	36.68	336.42		247.33	43.36	45.54	336.23	
1960	251.46	53.28	33.54	338.28		251.46	44.08	57.40	352.94	
1961	258.67	54.81	28.86	342.34		258.67	45.34	59.59	363.60	
1962	247.50	52.44	34.85	334.79		247.50	43.39	72.49	363.38	
1963	249.99	52.97	40.90	343.86		249.99	43.82	73.38	367.19	
1964	248.68	52.70	46.94	348.32		248.68	43.59	94.12	386.39	
1965	290.90	61.64	53.38	405.92		290.90	50.99	108.01	449.90	
1966	300.86	63.75	66.56	431.17		300.86	52.74	91.93	445.53	
1967	298.21	63.19	58.92	420.32		298.21	52.28	110.03	460.52	
1968	327.07	69.31	75.72	472.10		327.07	57.34	126.33	510.74	
1969	342.48	72.57	91.05	506.10		342.48	60.04	97.27	499.79	
1970	347.88	73.72	88.06	509.66		347.88	60.98	91.12	499.98	
1971	367.33	77.84	58.70	503.87		367.33	64.39	116.19	547.91	
1972	368.37	78.06	55.51	501.94		368.37	64.58	113.57	546.52	
1973	458.64	97.19	94.09	649.92		458.64	80.40	140.24	679.28	
1974	528.81	112.05	108.69	749.55		528.81	92.70	197.95	819.46	
					57.53 (depr.) 27.68 (int.) 36.25[a] (mgmt.) 119.66 (total)					54.67 (depr.) 27.68 (int. @ 7%) 36.25[a] (mgmt.) 113.60 (total)

[a] Management charge is figured as 2.5 percent of revenue (25 tons per acre times $50 per ton).

TABLE C9

Indexes[a] of Input Prices and Total Non-Raw-Product Costs to Process Raw Tomatoes into Bulk-Packed Tomato Paste (32 percent), California, 1951-1974, Assuming 1974 Production Technology

Input / Year	(1) Labor	(2) Drum	(3) Electricity	(4) Gas	(5) Hauling	(6) Total non-raw product cost
original units	hour	drum	KWH	therm	ton	dollars per ton
	1967=100					
1951	53.96	72.05	101.74	62.32	45.80	61.80
1952	55.40	72.59	102.57	64.18	48.09	63.00
1953	57.91	75.74	102.78	66.72	50.49	65.80
1954	58.99	76.26	105.58	67.33	53.01	67.00
1955	61.87	79.18	100.60	69.07	55.66	69.50
1956	65.47	84.85	97.69	71.44	58.44	73.60
1957	67.63	92.53	99.04	71.86	61.36	78.30
1958	69.78	98.80	99.70	76.07	64.43	80.40
1959	73.02	95.83	100.10	82.95	67.55	83.40
1960	77.34	91.72	101.19	87.21	71.03	83.60
1961	81.29	91.47	101.69	88.71	74.58	85.20
1962	84.17	91.47	102.08	89.15	78.31	86.70
1963	85.97	93.25	101.29	91.85	82.23	89.10
1964	87.44	94.12	100.40	90.73	86.34	90.70
1965	91.48	96.44	100.10	92.82	90.66	93.80
1966	94.17	98.31	99.60	96.71	95.19	96.60
1967	100.00	100.00	100.00	100.00	100.00	100.00
1968	101.67	103.65	100.89	92.67	104.81	102.50
1969	108.65	107.68	101.99	93.27	103.94	105.30
1970	124.92	114.43	104.77	103.29	103.06	111.90
1971	134.95	120.50	113.60	108.00	114.00	119.80
1972	144.65	127.20	121.50	114.10	112.04	124.30
1973	154.84	129.20	129.30	126.70	130.63	133.50
1974	174.19	148.91	155.57	157.47	138.07	151.20

[a] Tomato paste input price and per-ton raw equivalent processing cost series have not been reported in actual dollars to protect the confidentiality of the cooperative which assisted us in this study. Those interested in obtaining access to this data may contact Tri/Valley Growers, San Francisco, California.

Sources by column:

(1) 1951-1963: U.S. Dept. of Labor, _Employment and Earnings, States and Areas_ [BLS (SIC 2032-5,7].
1964-1974: Tri/Valley Growers.
(2) 1951-1953: U.S. Dept. of Labor, _Wholesale Price Indexes_, "Metal Containers."
1954-1974: _Ibid._, "Barrels, Drums, and Pails."
(3) 1951-1957: _Ibid._, "Electricity."
1958-1974: _Ibid._, "Electrical Power."
(4) 1951-1957: _Ibid._, "Gas."
1958-1974: _Ibid._, "Gas Fuels."
(5) 1951-1965: Discount 1966 value by 7 percent annually: California Trucking Association estimate.
1966-1974: Tri/Valley Growers.

electricity and natural gas. Index series used were "electrical power" and "gas fuels" for 1958-1974 and "electricity" and "gas" for 1951-1957. These are reported on Table C9, columns (3) and (4). The 1958-1974 series for both goods reflect prices at different points in the transmission system than the 1951-1957 series. Thus, 1957-1958 price changes reflect these alterations in series definitions.

Tomato Hauling

An index of 1966-1974 tomato hauling costs per barrel of paste was provided by the cooperative under study. These were average rates for all tomato canneries. For the period 1951-1965, an index of annual ton-mile hauling costs by truck was sought for processing fruits and vegetables. But no truck rate series of any sort could be uncovered without lengthy digging in Public Utilities Commission files, where records of charges by regulated carriers are kept.1/ Acting on the statement of a PUC official that hauling rates in the 1950's increased an average of 5 percent annually, the 1965-1974 series was moved back to 1951 by 5 percent annual increments. This process negatively biases the variance of hauling rates because the series 1951-1964 has no variance around its 5 percent trend. The trend variance of total nontomato costs of bulk paste are therefore slightly negatively biased from this source.

Total Cash Costs

An index series of total processor nontomato cash costs is reported in Table 3.1, Section IV, column (9).

1/ Suitable indexes of general transportation rates were also unavailable.

APPENDIX D

Procedures for Constructing Historical Series of Revenue Variables

U.S. Wholesale Prices of Consumer-Size Tomato Sauce (MP^{tr}_{sce})

The procedure used to construct a tomato sauce price series was as follows:

(a) The average realized price of catsup, f.o.b. plant, California, in 24/14 oz. cases was converted to an index in Table D1, columns (1) and (2).

(b) The April, 1975 retail price of 32 oz. jar spaghetti sauce produced by the distributor studied was obtained at a supermarket. Typical canned tomato markups at retail (21 percent) and wholesale (14 percent), as reported in National Commission on Food Marketing and U.S.D.A., Market and Transportation Situation, were applied to this figure to estimate the 1975 price per 32 oz. jar of spaghetti sauce at plant. Assuming sauce and catsup prices have moved closely together, the 1975 sauce price was then multiplied by the catsup price index series to obtain a historical spaghetti sauce series, Table D1, column (3). Since the purpose is to estimate the probability moments of sauce prices, it makes no difference whether this sauce was sold in its present form in 1951. It is sufficient to know how its prices would have behaved had it been produced then in its present form. Such information is assured by the assumption of similarity between catsup and sauce prices because it is known that the catsup price series refers to a standard product.

(c) The next step was to calculate the transformation represented by the coefficient y in the distributor and cooperative profit functions, Tables 3.2 and 3.3. This transformed series expresses distributor sales revenue per

TABLE D1

Procedure for Calculating Tomato Sauce Revenue of the Distributor/Reprocessor

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Fiscal year	Average price of catsup, F.O.B. plant, California	Index of catsup prices	Imputed price of spaghetti sauce, sauce plant	Imputed price of spaghetti sauce, sauce plant	Imputed price of spaghetti sauce, sauce plant	Imputed value of spaghetti sauce, sauce plant	Imputed value spaghetti sauce, sauce plant, yield constant
	dollars per 24/14 oz glass fancy	1967=100	dollars per 32 oz jar	dollars per 55 gal barrel (32%)	dollars per ton raw (MP^{tr}_{sce})	dollars per acre ($\tilde{MV}^{a}_{sce}$)	dollars per acre (MV^{a}_{sce})
1951-52	3.57	86.23	.2020	432.280	258.850	4,519.52	5,088.99
1952-53	3.04	73.43	.1720	368.080	220.407	3,588.23	4,333.20
1953-54	3.20	77.29	.1811	387.554	232.068	4,534.61	4,562.46
1954-55	3.39	81.88	.1918	410.452	245.780	4,269.23	4,832.03
1955-56	3.75	90.58	.2122	454.108	271.921	4,532.92	5,345.97
1956-57	3.55	85.75	.2009	429.926	257.440	4,500.05	5,061.27
1957-58	3.32	80.19	.1879	402.106	240.782	3,891.04	4,733.77
1958-59	3.16	76.33	.1788	382.632	229.121	4,098.97	4,504.52
1959-60	3.20	77.29	.1811	387.554	232.068	3,367.31	4,562.46
1960-61	3.34	80.68	.1890	404.460	242.192	4,318.28	4,761.49
1961-62	3.44	83.09	.1947	416.658	249.496	3,924.57	4,905.09
1962-63	2.98	71.98	.1686	360.804	216.050	4,191.37	4,247.54
1963-64	3.13	75.60	.1771	378.994	226.942	4,609.19	4,461.68
1964-65	3.36	81.16	.1902	407.028	243.729	5,591.14	4,791.71
1965-66	3.75	90.58	.2122	454.108	271.921	5,696.74	5,345.97
1966-67	3.86	93.24	.2185	467.590	279.994	5,574.68	5,504.68
1967-68	4.14	100.00	.2343	501.402	300.241	5,311.26	5,902.74
1968-69	3.98	96.14	.2253	482.142	288.708	6,415.09	5,676.00
1969-70	4.09	98.79	.2315	495.410	296.653	6,642.06	5,832.20
1970-71	4.30	103.86	.2433	520.662	311.774	7,460.75	6,129.48
1971-72	4.39	106.04	.2485	531.790	318.438	7,419.61	6,260.49
1972-73	4.60	111.11	.2603	557.042	333.558	8,495.72	6,557.75
1973-74	4.97	120.05	.2813	601.982	360.468	8,110.53	7,086.80
1974-75	7.22	174.40	.4086	874.404	523.595	12,519.16	10,293.88

Sources by column:

(1) *Food Production/Management*.
(2) Column (1) ÷ 4.14.
(3) Imputed 1975 price per pound F.O.B. sauce plant ($.4086/lb.) is divided by 174.40 to obtain 1967-68 base year value $.2343/lb. Remaining series constructed by multiplying .2343 by column (2). Imputed price $.4086 calculated from retail shelf prices and assumed markups reported in National Commission on Food Marketing and *Marketing and Transportation Situation*.
(4) Column (3) times 2140. Factor 2140 obtained as follows: (a) 55 gals paste (32% solids) = 535 lbs; (b) 535 x .32 = 171.2 lbs solids; (c) 171.2 x 12.5 = 2140 lbs sauce @ 8% solids.
(5) Column (4) ÷ 1.67. Factor 1.67 is tons tomatoes per 55 gal barrel 32% paste, when tomatoes average 5.46% solids.
(6) Column (5) times six-county yields (column 2, Table 3.1).
(7) Column (5) times 19.66 (mean of six-county yields, 1950-1974).

unit of bulk paste sold to it by the cooperative (Table D1, column (4)). Calculation of the transformation factor y is explained on Table D1, footnote 4.

(d) As explained in Section II, the cooperative is thought for our purposes to sell bulk paste in terms of equivalent tons of raw tomatoes. This facilitates calculation of cooperative net margin since coop inputs are also expressed in these terms; it also simplifies calculation of the sales-minus price paid to growers, which is expressed in tons of raw tomatoes. Column (5), Table D1, records the imputed historical prices of sauce per ton equivalent raw tomatoes (MP^{tr}_{sce}).

When column (5) is multiplied by 19.66, the historical mean of six-county tomato yields, one obtains column (7), the per-acre market value of tomato sauce (MV^{a}_{sce}) which, however, contains no random yield component. Column (7) is appropriate for estimating the probability moments of cooperative sales-minus sales revenue, provided the coop contracts to sell a fixed number of tons of bulk paste. This is because sales revenue from a tonnage contract will not vary with yield fluctuations. If instead the coop contracts to sell all the bulk paste produced from a certain number of acres, sales revenue is best expressed by multiplying MP^{tr}_{sce} by random yields, i.e., the actual historical yield series. That is accomplished in column (6).

U.S. Wholesale Prices of California Bulk-Packed Tomato Paste (MP^{tr}_{pst})

A collection of tomato paste price series from a number of sources, and for a number of container sizes and percent solids, is found in Table D2. Figures in parentheses at column headings are tomato solids as percentages of total weight. These series were compiled by recording weekly price quotes and taking their simple averages over fiscal years. The averages are not

TABLE D2

Annual Average Tomato Paste Prices, California, 1951-1974, by Container Size and Percent Solids

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Fiscal year	48/6 oz	96/6 oz	6/10's[a] (26%)	6/10's[b] (26%)	6/10's (32%)	55-gal drums (32%)	Ratio column (6) to column (3)	Ratio column (6) to column (2)	Ratio column (6) to column (5)
	dollars per case					$/drum			
1951-52	4.375	7.487	6.852						
1952-53	3.843	6.695	5.848						
1953-54	3.737	6.128	5.523	5.51					
1954-55	3.845	6.968	5.818	5.80					
1955-56	3.835	7.273	7.064	7.32					
1956-57	3.500	7.057	7.276	6.99					
1957-58	3.480	7.100	6.004	5.74					
1958-59	-	7.090	5.345	5.05					
1959-60	-	6.870	5.493	5.53					
1960-61	-	7.840	6.523	6.63					
1961-62	-	8.694	7.589	7.75	9.052				
1962-63	-	7.196	6.208	6.00	-				
1963-64	-	8.161	6.412	6.11	7.990	106.380	16.592	13.035	13.315
1964-65	-	8.219	6.404	6.53	7.948	107.800	16.834	13.116	13.564
1965-66	-	9.656	8.646	8.75	9.886	146.600	16.956	15.182	14.830
1966-67	-	9.990	9.361	9.63	10.806	125.050	13.359	12.518	11.572
1967-68	-	11.125	10.875	10.88	12.183	163.850	15.067	14.728	13.449
1968-69	-	10.657	8.641	8.31	11.338	122.591	14.330	11.618	10.921
1969-70	-	10.600	6.973	6.68	9.350	89.796	12.922	8.500	9.637
1970-71	-	10.119	7.404	7.53	9.521	94.126	12.323	9.017	9.583
1971-72	-	10.477	8.152	8.49	9.700	107.325	12.437	9.677	10.453
1972-73	-	11.081	8.627	-	?	113.522	12.289	9.568	-
1973-74	-	12.021	9.910	-	?	127.615	12.210	10.066	-
1974-75	-	-	-	-	-		-	-	-

[a] Averages of weekly prices.

[b] Series reported in King, Jesse, and French, averaging quarter-ending prices only.

Sources by column:

(1) Commercial Bulletin. Percent solids were not reported.
(2) Canning Trade. Percent solids were not reported.
(3) Pacific Fruit News: averages from weekly data (52 weeks). This series was calculated by the author.
(4) Pacific Fruit News: averages from quarter-ending weekly data (4 weeks).
(5) Pacific Fruit News: weekly averages.
(6) Tri/Valley Growers.
(7)-(9) Derived.

weighted by seasonal changes in shipment volumes. For comparison, the price series for 6/10's, 26 percent, reported in King, Jesse, and French, pp. 121-122, is listed alongside our own series for this product. The two series differ because of our series averages price quotes from each week of the year whereas the King, Jesse, and French series averages price quotes from the first week of January, April, July, and September only.

Since the present study's purpose is to estimate the probability of prices of 32 percent tomato paste packed in 55-gallon drums, it would be possible to go immediately to column (6) for this purpose. The sample size for this series, however, is uncomfortably short to accommodate statistical tests for the price predictive power of another variable, such as time or inventory levels. Thus column (6) has been extended backwards in time by establishing a statistical correspondence between bulk paste prices and prices of another case size with longer history. Since bulk paste, especially in containers larger than #12's (gallon), were not in general use during the 1950's, the result of this extension is a series of hypothetical prices for bulk paste had such container sizes been generally used. This technique is legitimate when the purpose is to estimate a probability distribution for use in predicting future price variability, providing the statistical correspondence used to extend the series is valid and that past price fluctuations or cyclical behavior will be repeated in the future.

In order to extend the 55-gallon price series backward, the ratio series of 55-gallon to 6/10, 26 percent prices was regressed against time. The regression results were

$$\begin{aligned} Y &= 17.28 - .526T \\ &\quad (30.03)\ (-6.21) \end{aligned}$$

where Y is the ratio series and T is time. The t-values, given in parentheses, indicate a stable relationship between the two price series composing Y.

Employing the ratio column (6)/column (3) = 17.28, therefore, and observing an annual trend in this ratio of -.526, column (6) in Table D2 was extended back from 1963-1964 to 1951-1952. The extended series is given in column (1), Table D3.

It is a simple procedure from this point to express prices of 55-gallon, 32 percent paste in terms of equivalent tons raw tomatoes. Approximately .5988 of a 55-gallon barrel of paste, which weighs approximately 535 pounds, is produced from one ton of tomatoes containing 5.46 percent tomato solids. The reciprocal of .5988 is therefore the paste-to-raw-tomato conversion factor x as represented in the cooperative and distributor profit equations, Tables 2.2 and 2.3. Multiplying the .5988 transformation factor by prices per 55-gallon barrel, one obtains prices per equivalent ton tomatoes (MP^{tr}_{pst}) as reported in column (2), Table D3. Acre-value series are then formed in column (3), (4) as in Table D1.

Nontomato Farmer Revenue ($REVF^{a}_{ntom}$)

A simplified nontomato crop pattern for Central Valley tomato growers has been formulated with acreage weights .5 for corn, .3 for wheat, .2 for beans. The exact nontomato acreage is not important at this point because this can be varied in the main expected utility and E-V programs.

Table D4 records price and yield histories of these three crops in California. Wheat and bean price series represent the price per bushel or hundredweight that was realized at farm. Corn prices are quoted at Stockton. There was little available data on pink beans as such; resort was made to "all bean" prices and "all nonlima bean" yield reports. For each crop a revenue-per-acre series is calculated by multiplying price per unit times unit yield per acre. Employing the crop weights listed above, a weighted average revenue per acre was then formed (column 10).

TABLE D3

Computation Procedure Used to Estimate Average Annual Market Value of Bulk-Packed Tomato Paste Produced from One Acre of Processing Tomatoes, California, 1951-1973

Year	(1) Market price and imputed market price 55 gal paste (32%)	(2) Market price of 55 gal paste (32%), per ton raw equivalent (5.46% solids)	(3) Market value[a] of 55 gal paste produced from one acre (actual six-county yields)	(4) Market value[b] of 55 gal paste produced from one acre (average six-county yields)
	dollars per barrel	dollars per ton raw	dollars per acre	
1951	113.678	68.070	1,188.50	1,338.26
1952	97.019	58.095	945.79	1,142.15
1953	91.636	54.872	1,072.20	1,078.78
1954	96.529	57.802	1,010.38	1,136.39
1955	117.199	70.179	1,169.88	1,379.72
1956	120.709	72.280	1,263.45	1,421.02
1957	99.614	59.649	963.93	1,172.70
1958	88.678	53.100	949.96	1,043.95
1959	91.136	54.572	791.84	1,072.89
1960	108.220	64.802	1,155.42	1,274.01
1961	125.911	75.395	1,185.96	1,482.27
1962	103.000	61.676	1,196.51	1,212.55
1963	106.380	63.700	1,293.75	1,252.34
1964	107.800	66.551	1,526.68	1,308.39
1965	146.600	87.784	1,839.07	1,725.83
1966	125.050	74.880	1,490.86	1,472.14
1967	163.850	98.113	1,735.62	1,928.90
1968	122.591	73.407	1,631.10	1,443.18
1969	89.796	53.770	1,203.91	1,057.12
1970	94.126	56.363	1,348.77	1,108.10
1971	107.325	64.266	1,497.40	1,263.47
1972	113.522	67.977	1,731.37	1,336.43
1973	127.615	76.416	1,719.36	1,502.34

[a] $\tilde{MV}^{A}_{sce}$

[b] MV^{A}_{sce}

Sources by column:

(1) 1951-1962: derived from index of prices of 6/10 paste, 26%, Table D2, column (3).
1963-1974: Tri/Valley Growers.
(2) Column (1) times .5988, the number of barrels of 32% tomato paste produced from one ton of tomatoes at 5.46% solids.
(3) Column (2) times six-county yields, Table 3.1, column (2).
(4) Column (2) times 19.66 (1951-1974 mean of six-county yields).

TABLE D4

Average Annual Prices, Yields, and Per-Acre Revenues Earned from Corn, Wheat, and Dry Edible Bean Cultivation, California, 1951-1974

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	Corn			Wheat			Beans					
Year	Price	Yield	Revenue	Price	Yield	Revenue	Price	Yield	Revenue	Wght Ave revenue per acre ($REVF^{A}_{NTOM}$)	Wght Ave cash cost per acre (VCF^{A}_{NTOM})	Wght Ave fixed cost per acre (FCF^{A}_{NTOM})
	dollars per bushel	bushels per acre	dollars per acre	dollars per bushel	bushels per acre	dollars per acre	dollars per cwt	cwt per acre	dollars per acre	dollars per acre		
1951	2.26	34	76.84	2.23	17	37.91	9.00	13.41	120.69	73.93		
1952	2.03	35	71.05	2.25	21	47.25	10.10	12.55	126.75	75.05		
1953	1.98	36	71.28	2.12	19	40.28	9.70	13.77	133.57	74.44		
1954	1.93	48	92.64	2.15	20	43.00	9.00	13.29	119.61	83.14		
1955	1.85	66	122.10	2.06	21	43.26	8.20	11.96	98.07	93.63		
1956	1.68	67	112.56	2.00	21	42.00	8.00	13.11	104.88	89.86		
1957	1.57	74	116.18	2.11	22	46.42	8.40	12.21	102.56	92.53		
1958	1.54	73	112.42	1.80	22	39.60	8.50	12.58	106.93	89.48		
1959	1.52	73	110.96	1.79	23	41.17	8.90	13.06	116.23	91.08		
1960	1.48	75	111.00	1.80	22	39.60	10.10	12.26	123.83	92.15		
1961	1.50	72	108.00	1.96	27	52.92	9.80	12.93	126.71	95.22		
1962	1.54	75	115.50	2.01	30	60.30	8.90	13.36	118.90	99.62		
1963	1.58	77	121.66	1.90	24	45.60	9.50	14.73	139.93	102.50		
1964	1.61	83	133.63	1.45	26	37.70	9.60	13.75	132.00	104.52		
1965	1.58	89	140.62	1.46	26	37.96	11.30	13.45	151.98	112.09		
1966	1.60	92	147.20	1.56	27	42.12	10.70	13.78	147.45	115.73		
1967	1.45	84	121.80	1.45	33	47.85	14.20	12.68	180.05	111.26		
1968	1.51	95	143.45	1.36	33	44.88	11.00	13.67	150.37	115.26		
1969	1.59	92	146.28	1.45	34	49.30	10.19	11.30	115.15	110.96		
1970	1.73	98	169.54	1.43	45	64.35	10.14	13.20	133.84	130.84		
1971	1.66	94	156.04	1.63	47	76.61	11.30	13.20	149.16	130.83		
1972	1.75	100	175.00	1.72	48	82.56	12.79	18.00	230.22	158.31		
1973	2.47a/	109	269.23	2.71a/	49	132.79	33.25	19.50	648.37	304.13		
1974	3.47	107c/	343.47	4.24b/	48	158.88	32.17	14.43	464.21	312.24	219.99	36.30

a/ First 8 months only. b/ June, 1974. c/ Early estimate.

Sources by column:

(1) Agricultural Statistics, Stockton price, #2 yellow.
(2) Op. cit., California yields.
(3) Column (1) times column (2).
(4) Agricultural Statistics, average California winter wheat price.
(5) Op. cit., California yields. For 1973-1974, see "Crop Production: Wheat," SRS, USDA, 2/23/75.
(6) Column (5) times column (6).
(7) Agricultural Statistics, "All Beans, California." For 1969-1974 see "California Market Summary," Federal-State Market News Service.
(8) Op. cit., "Other Beans, California." For 1969-1974, see "Annual Dry Bean Summary," California Crop and Livestock Reporting Service.
(9) Column (7) times column (8).
(10) The weights employed are: Corn, .5; Wheat, .3; and Beans, .2.
(11)(12) Agricultural Extension Service, 1974, cost study for Sacramento Valley.

Note on Price Deflation

A true measure of financial risk must isolate inflationary from real sources of random fluctuation. Expected levels and fluctuations of a price variable which are due to expected levels and fluctuations of all prices are not a concern to the business decision maker because these are purely nominal effects and do not represent changes in real value or real profit. The temporary state of the economy in 1975 in which inflation was associated with a fall in average real income does not contradict this statement; any historical or prospective change in real national income which influences real changes in our price variables is *ipso facto* included in this analysis. An immediate advantage of real price series is that spurious correlations are avoided between variables that are not intrinsically related but participate only in the same general price movements.

Real price series have been calculated in this study by dividing all price series by the Wholesale Price Index reported by the U.S. Department of Labor. The wholesale index was preferred to the retail because prices analyzed in this study occur at wholesale market levels. The wholesale index series was converted to a base 1974=100 in order to render the current decision situation more meaningful.

REFERENCES

Arrow, K. J. "The Theory of Risk Aversion." Essays in the Theory of Risk Bearing. North-Holland Publishing Company, Amsterdam, 1971.

Bauer, P. T. and F. W. Paish. "The Reduction of Fluctuations in the Incomes of Primary Producers." Economic Journal 62:248, 1952.

Buccola, S. T. Portfolio Evaluation of Long-Term Marketing Contracts for U.S. Farmer Cooperatives. Unpublished Ph.D. dissertation, University of California, Davis, 1976.

California Agricultural Labor Commission. Report and Recommendation of the Agricultural Labor Commission. Sacramento, 1963.

California Department of Agriculture, Agricultural Commissioners' Offices. Agricultural Crop Reports, annual issues by county.

California Department of Agriculture, Crop and Livestock Reporting Service. Annual Dry Bean Summary. Sacramento, annual issues.

California Department of Employment, Division of Farm Labor Service. California Weekly Farm Labor Report. Report 881-A, mid-August issues.

Canning Trade. Canning Trade, Inc., Baltimore, Maryland, weekly issues. (From 1971, issued monthly as Food Production/Management.)

Chiang, A. C. Fundamental Methods of Mathematical Economics. McGraw-Hill, New York, 1967.

Commercial Bulletin. Fred Bailey, publisher. Los Angeles, California, weekly issues. (Now issued monthly as Grocery Bulletin.)

Cools, P. W. "Decentralization and the Transfer Price Problem." Journal of Business 28:2, 87-94, 1955.

Council of Economic Advisors. Economic Indicators. Government Printing Office, Washington, D.C., monthly issues.

Cutler, L. and D. S. Pass. A Computer Program for Quadratic Mathematical Models to be Used for Aircraft Design and Other Applications Involving Linear Constraints. Rand Report R-516-PR, Santa Monica, California, 1971.

Dean, J. "Decentralization and Intra-Company Pricing." Harvard Business Review 33:4, 1955.

Feldstein, M. S. "Liquidity Preference and Portfolio Selection." Review of Economic Studies 36:105, 1969.

Ferguson, C. E. "Theory of Multidimensional Utility Analysis in Relation to Multiple Goal Business Behavior: A Synthesis." Southern Economic Journal 32:2, 1965.

Freund, J. E. Mathematical Statistics, 2nd edition. Prentice-Hall, Inc., Englewood Cliffs, 1971.

Freund, R. J. "The Introduction of Risk Into a Programming Model." Econometrica 24:3, 253-263, 1956.

Garoyan, L. "Producer-First Handler Exchange Arrangements for Selected California Processed Fruits, Vegetables, and Nuts." Coordination and Exchange in Agricultural Subsectors. North Central Regional Publication, Monograph NC-117, 1975.

Gould, J. R. "Internal Pricing in Firms Where There Are Costs of Using An Outside Market." Journal of Business 37:1, 1964.

Haegert, L. and R. Harris. Kelley's Cutting Plane Algorithm. Computer Center, University of California, Berkeley, 1968.

Halter, A. N. and G. W. Dean. Decisions Under Uncertainty. Southwestern Publishing Co., Cincinnati, 1971.

Hampton, J. M., P. G. Moore and H. Thomas. "Subjective Probability and Its Measurement." Journal of the Royal Statistical Society Series A, 136:1, 1973.

Hirshleifer, J. "On the Economics of Transfer Pricing." Journal of Business 29:3, 1956.

Johnston, J. Econometric Methods, 2nd edition. McGraw-Hill, New York, 1972.

King, G. A., E. V. Jesse and B. C. French. Economic Trends in the Processing Tomato Industry. Department of Agricultural Economics, University of California, Davis, Report 73-4, 1973.

Kmenta, J. Elements of Econometrics. Macmillan Publishing Co., New York, 1971.

Luce, R. D. and H. Raiffa. Games and Decisions. John Wiley & Sons, New York, 1957.

Markowitz, H. M. Portfolio Selection. John Wiley & Sons, New York, 1959.

Mehren, G. L. How Is the Market Made in Integrated Industries? Address to National Institute of Animal Agriculture, Purdue University, Lafayette, Indiana, 1958.

National Commission on Food Marketing. Food from Farmer to Consumer. Washington, D.C., 1966.

Pacific Fruit News. R. Hitchcock, publisher. San Jose, California, weekly issues.

Pratt, J. W. "Risk Aversion in the Small and in the Large." Econometrica 32:1-2, 1964.

Reed, A. D. Machinery Costs and Performance. Agricultural Extension Service, University of California, Davis, 1972.

Touche-Ross, Inc. Accounting and Taxation for Cooperatives. Revised (no place given), 1974.

University of California, Agricultural Extension Service. Sample Costs of Processing Tomato Production, issued irregularly by county.

U.S. Department of Agriculture. Agricultural Statistics. Washington, D.C., 1973.

U.S. Department of Agriculture and California Department of Agriculture, Federal-State Market News Service. California Market Summary: Dry Beans. Sacramento, annual issues.

U.S. Department of Agriculture, Economic Research Service. Farm Real Estate Market Developments. Washington, D.C., monthly issues.

U.S. Department of Agriculture, Economic Research Service. Marketing and Transportation Situation, monthly issues.

U.S. Department of Agriculture, Statistical Reporting Service. Agricultural Prices. Washington, D.C., annual summaries.

U.S. Department of Agriculture, Statistical Reporting Service. California Vegetable Crops. Sacramento, annual issues.

U.S. Department of Agriculture, Statistical Reporting Service. Crop Production: Wheat. Washington, D.C., annual issues.

U.S. Department of Agriculture, Statistical Reporting Service. Vegetables-Processing. Washington, D.C., annual summaries.

U.S. Department of Labor, Bureau of Labor Statistics. Employment and Earnings, States and Areas, 1939-1972. Bulletin 1370-10, 1974.

U.S. Department of Labor, Bureau of Labor Statistics. Wholesale Prices and Price Indexes, annual summaries and selected monthly issues.

Von Neumann, J. and O. Morgenstern. Theory of Games and Economic Behavior. Princeton University Press, Princeton, 1947.

Wiens, T. B. "Peasant Risk Aversion and Allocative Behavior: A Quadratic Programming Experiment." American Journal of Agricultural Economics 58:4, 1976.

Williams, T. T. Contract Vegetable Marketing. Southern University Bulletin 4, Scotlandville, Louisiana, 1961.